Estimating the Richardson-Zaki Index *n* for Non-Spherical Irregular Natural Sediments in a Concentrated Suspension

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***Abstract* --- The hindered settling velocity (u_h) of the particles is considered as a part of the terminal (free) settling velocity (u_t). Richardson and Zaki investigated this relation and derived an equation known as R & Z formula ($u_h = u_t \varepsilon^n$) for spherical particles. Currently work is modified the R & Z index *n* to become suitable for non-spherical irregular natural sediments that settled in concentrated suspension known as palm oil mill effluent (POME). Aspect ratio (AR) adopted as a shape factor to determine the particle diameter. The *n*-indexes modified at different Reynolds number (Re) taking into consideration the wall effect (λ). The obtained results shows that the index *n* for non- spherical irregular particles has close values of index *n* for the spherical particles in comparison to the previous published data.**

Keywords:*Palm oil mill effluent , Hindered settling velocity, n-index, Reynold Number, Aspect Ratio*

I. INTRODUCTION

Hindered settling occurs in a concentrated suspension when the particles become closer then get sufficient interaction forces that hinder the settling of neighbouring particles. A complementary effect causes fluid displacement because of the settling particles that produced up flow force through the inter-particle voids [1]. The other name of this type of settling is "zone settling or mass settling" because there will be sharp boundary between the settling suspension (bottom layer) as a mass and supernatant (upper layer) as a clear layer [2, 3]. Baldock et al. (2004) concluded that at high sediment concentration, the particle settling velocity will be reduced to some small fraction of their velocity in the clear water [4]. Furthermore, the hindered settling velocity of suspension of multisize particles is more difficult and complex than the single particle size suspension. This is because the settling velocity of each particle size is affected to different degrees by the counter flow of the displaced fluid due to the settling of other components [5].

The hindered settling velocity is usually considered as a part of the discrete (free) settling velocity [6]. This has been investigated extensively by Richardson and Zaki and derived R & Z equation [7]:

$$u_h = u_t \varepsilon^n \quad (1)$$

$$u_h = u_t (1 - C)^n \quad (2)$$

where; the hindered settling (u_h) of uniform particles is equal to terminal settling velocity (u_t) of single particle times the correction factor equal to the voidage (ε) raised to power (n) called Richardson and Zaki index and (C) is the fractional volumetric concentration of solid particle in the suspension (the volume of particles divided by the total volume of the suspension) [4, 8, 9].

The index n is an empirical parameter that depends on single particle Reynolds number and the ratio of the particle diameter (d_p) to the column diameter (D) known as λ. Tables I and II give the different values of index n without and with wall effect respectively [10-12].

TABLE I. RICHARDSON-ZAKI INDEX *n* VALUES WITHOUT WALL EFFECT [10, 11]

Re at terminal velocity	**index *n* without wall effect**	**Equation number**
Re < 0.2	4.65	(3)
0.2 < Re < 1	$4.35\ \mathrm{Re}^{-0.03}$	(4)
1 < Re < 500	$4.45\ \mathrm{Re}^{-0.10}$	(5)
500 < Re	2.39	(6)

TABLE II. RICHARDSON-ZAKI INDEX *n* VALUES WITH WALL EFFECT [12]

Re at terminal velocity	**index *n* with wall effect**	**Equation number**
Re < 0.2	$n = 4.65 + 19.5\,(\frac{d_p}{D})$	(7)
0.2 < Re < 1	$n = (4.35 + 17.5\,(\frac{d_p}{D}))\,\mathrm{Re}^{-0.03}$	(8)
1 < Re < 500	$n = (4.45 + 18\,(\frac{d_p}{D}))\,\mathrm{Re}^{-0.1}$	(9)
500 < Re	2.39	(10)

Furthermore, the Richardson and Zaki index, n, has been modified by Khan and Richardson in 1989 as follow [5, 13]:

$$\frac{4.8-n}{n-2.4} = 0.043\,Ga^{0.57}\left[1-1.24\left(\frac{d_p}{D}\right)^{0.27}\right] \quad (11)$$

They used Galileo number, Ga, because it can be directly determined from the particle and fluid properties, unlike Reynolds number expression that has particle terminal settling velocity.

Several researchers have focused on the effect of uniform particle concentration on the sedimentation rate of the suspension and found that the hindered settling rates decrease with the increase in particle concentration [7, 14]. Chong et al. (1979) used different uniform particle shapes such as spherical glass beads, cubical crystals, plastic pellets, brick like sugar crystals and angular mineral crystal with different column diameters to determine the wall effect. It was found that R & Z equation (Equation 2) can be correlated with the constant settling velocity for each particle size and shape over voidage range (0.65-0.9) and the wall effect is considered smaller than that predicted by Richardson and Zaki [15]. Baldock et al. (2004) used artificial and natural sands at high concentration to determine their settling velocity and compared it with R & Z equation. The aim of their study was to develop a straightforward method to get an appropriate value of index n that could be applied to natural beach sand. Finally, they concluded that at high sediment concentrations the hindered settling velocity was reduced to some small fraction of their free settling velocity. This hindered settling velocity can be accurately predicted by the R & Z equation with providing an appropriate value for index n for particular material [4].

Another study mentioned that it remains uncertain, however, whether the range of the exponent n that proposed by Richardson and Zaki (1954) can be applied directly to cohesive mud flocs, which differ from non-cohesive (sand) particles in many ways (e.g. irregular shapes; wider distributions of size and density; structural variations) [16]. But Tomkins et al. (2005) showed that the R & Z equation is still useful to describe the sedimentation rate of irregular sand grains in the hindered settling regime[9]. However, it should be included in an appropriate sedimentation exponent. Due to that, this equation had been adopted in this study.

This paper is trying to modify the R & Z index n at different Reynolds number (Re) "taking into consideration the wall effect (λ)" to become suitable for non-spherical irregular natural sediments that settled in concentrated suspension known as palm oil mill effluent (POME) which has 19,500mg/L total suspended solid (TSS). In addition, since the POME particles are nons-pherical and irregular shape, the aspect ratio (AR) adopted as a shape factor to determine the particle diameter.

II. MATERIALS AND METHOD

The concentrated suspension in this study is raw palm oil mill effluent (POME). POME was collected from palm oil mill factory in Pulau Pinang-Malaysia. Samples were stored in a sealed plastic container and preserved at a temperature less than 4^{O}C but above the freezing point in order to prevent the wastewater from undergoing biodegradation due to microbial action. The average physio-chemical properties of these samples are summarized in Table III.

Table II1. PHYSIO-CHEMICAL PROPERTIES OF RAW POME

Parameters	**Values**
Chemical Oxygen Demand (COD)	67,375 mg/L
Biological Oxygen Demand (BOD)	35,600 mg/L
Total Suspended Solid (TSS)	19,500 mg/L
pH	3.8

To get non-oily POME particles from raw POME, the raw POME was settling down naturally for 24 to 48 hours then the POME sludge (at the bottom) was draw by pump and dewatered using fibre materials with pore openings < 20 µm. Then this dewatered POME sludge was freeze dried completely by freeze dryer (Model LABCONCO). After that, the Soxhlet extraction by hexane was adopted to extract all the oil from the POME sludge to get non-oily particles. The sieve shaker (Model Retsch - AS 200) was used to determine the POME particle size. A series of sieves were arranged from the top to bottom with aperture openings as follows: 1400, 1000, 800, 500, 250, 125, 63, 45, 38, 20 µm and the final receiving pan at the bottom. The sieves were shaken horizontally for 20 min, followed by removing each sieve for reweighing and the weight of POME particles in each sieve was determined [17].

To find shape factor “Aspect ratio (AR)”, image analysis was carried out using OLYMPUS SZX9-stereo (or dissecting) microscope, equipped with a video camera, to determine Feret's diameter for particle sizes greater than 800 µm and OLYMPUS BX41-compound microscope for particle sizes less than 800 µm. The software used for this analysis is known as analySIS Image Processing "cell^A". AR is the ratio of the maximum Feret's diameter to the minimum Feret's diameter [18]:

$$\mathrm{AR} = d_{\mathrm{Feret\ max}} / d_{\mathrm{Feret\ min}} \qquad (12)$$

Non-oily POME particle densities were determined from its mass and volume. But it should be first determine the specific gravities of these particles by the following formula [19]:

$$Specific\,gravity,\,sp.gr. = \frac{(W_{2-}W_1)}{(W_2 - W_1) - (W_3 - W_4)} \qquad (13)$$

where W_1 is the empty weight of volumetric flask (g), W_2 is the weight of volumetric flask filled with POME sediments (g), W_3 is the weight of volumetric flask filled with distilled water and POME sediments (g) and, W_4 is the weight of volumetric flask filled with distilled water (g). The specific gravity of POME sediments was measured under room temperature 27 ±1 °C by water displacement method. A 25 mL volumetric flask was used in place of pycnometer to measure the volume of POME sediments. Boiled distilled water was used in this experiment to keep it free from dissolved gases which might hinder the settling of the POME particles in the flask [20]. After the determination of the specific gravity of POME particles, the density can be easily calculated by using the following formula:

$$Specific\,gravit,\,sp.\,gr. = \frac{density\,of\,\,the\,sample\,(g/cm^3)}{density\,of\,\,water\,\,(g/cm^3)} \qquad (14)$$

The water density depends on water temperature and atmospheric pressure. The water density at 28 °C is 0.9955 g /cm^3.

III. RESULTS AND DISCUSSION

The fractional volumetric (C) is one of the important parameter in this equation which is found by division of the volume of POME particles, mL ”= mass × density of POME particles, ρ ” by the total volume of POME sludge that was obtained from one Liter of raw POME after natural gravity settling “= 450 mL” . Table IV presents the fractional of non-oily POME particles for different particles sizes.

The index *n* of non-spherical irregular non-oily POME particles was modified depending on the R & Z equations for spherical particles that were presented earlier in Tables I and II (without and with wall effect respectively) [12].

TABLE IV. DETERMINATION OF FRACTIONAL VOLUMETRIC CONCENTRATION (C) OF THE NON-OILY POME PARTICLES AT DIFFERENT PARTICLES SIZES

Sieve size range (µm)	Mean diameter, d_{mean} (cm)	% wt Mass Retain	Particle density, ρ (g/cm^3)	Mass of POME particles = %wt × 3.9^a (g)	Volume of POME particles, (vol.) = mass × ρ (mL)	C = vol./450^b
- 45 + 38	0.00415	0.138	1.0410	0.5382	0.5170	0.0011
- 63 + 45	0.00540	0.375	1.0987	1.4625	1.3311	0.0030
- 125 + 63	0.01075	16.613	1.1506	64.7888	56.3087	0.1251
- 250 + 125	0.01875	23.025	1.3164	89.7975	68.2144	0.1516
- 500 + 250	0.0375	27.988	1.5118	109.1513	72.1995	0.1604
- 800 + 500	0.065	26.750	1.5900	104.3250	65.6132	0.1458
- 1000 + 800	0.09	4.425	1.6041	17.2575	10.7584	0.0239
- 1400 + 1000	0.12	0.675	1.6060	2.6325	1.6392	0.0036

a. The weight values of non-oily POME particles that were obtained from POME sludge after freeze dryer = 3.9 g

b. The total volume of POME sludge that was obtained from one Liter of raw POME after natural gravity settling = 450 mL

TABLE V. INDEX *n* AND HINDERED SETTLING VELOCITIES FOR NON-SPHERICAL NON-OILY POME PARTICLES USING ASPECT RATIO AS A SHAPE FACTOR AT DIFFERENT PARTICLE SIZES

Sieve size range (µm)	d_{mean} (cm)	AR	$d_{eq.}$ = d_{mean}×AR (cm)	Re	λ = d_{eq}/D*	*Without wall effect* $\lambda<0.02$ $n=4.65$ $Re<0.2$	*Without wall effect* $\lambda<0.02$ $n=4.35\,Re^{-0.03}$ $0.2<Re<1$	*Without wall effect* $\lambda<0.02$ $n=4.45\,Re^{-0.1}$ $1<Re<500$	*With wall effect* $\lambda>0.02$ $n=(4.45+18\,\lambda)Re^{-0.1}$ $1<Re<500$	Predicted Terminal Settling velocity (cm/s)	C	$u_h = u_t(1-C)^n$ (cm/s)
- 45 + 38	0.00415	2.845	0.012	0.019	0.00454	4.650	-	-	-	0.022	0.0011	0.0220
- 63 + 45	0.0054	2.632	0.014	0.067	0.00547	4.650	-	-	-	0.065	0.0030	0.0639
- 125 + 63	0.01075	2.546	0.027	0.378	0.01053	-	4.479	-	-	0.210	0.1251	0.1049
- 250 + 125	0.01875	2.062	0.039	1.076	0.01487	-	-	4.418	-	0.432	0.1516	0.1861
- 500 + 250	0.0375	2.038	0.076	4.123	0.02940	-	-	-	4.321	0.847	0.1604	0.3504
- 800 + 500	0.065	1.896	0.123	9.223	0.04739	-	-	-	4.247	1.178	0.1458	0.5300
- 1000 + 800	0.09	1.692	0.152	17.471	0.05856	-	-	-	4.135	1.587	0.0239	1.4356
- 1400 + 1000	0.12	1.270	0.152	17.524	0.05862	-	-	-	4.134	1.590	0.0036	1.5662
*Column diameter, D = 2.6 cm									$\sum u_t$=	5.931	$\sum u_h$=	4.2591

The index *n* equations for spherical particles were adjusted to become more suitable for non-spherical irregular particles. Some considerations were taken into account during the determination of index *n* such as the effect of multisize and non-sphericity of non-oily POME particles, the effect of different particle fractional volumetric concentration and the ratio between particle diameters (dp) to column diameter (D) that is expressed by λ. The λ ratio has directly proportional to the wall effect. When λ is less than 0.02 it can be assumed that there is no wall effect and when λ is greater than 0.02 it can be assumed that there is wall effect on the settling particles [19]. Furthermore, the values of λ and Re have very important roles to choose the suitable equation to determine index *n*. Tables V presents the index *n* and the hindered settling velocities (*uh*) of non-spherical non-oily POME particles Aspect Ratio (AR) as a shape factors. In addition, the predicted free settling velocity (cm/s) was determined by using the following equation:

Predicted free settling velocity, $u = (Re\ \mu) / (\rho d_{eq})$ (15)

where, Re=Reynold Number, μ= Supernatant POME viscosity at 28°C= 0.0138 g / (cm.s), ρ = Supernatant POME density =0.998 g /cm^3, and the equivalent diameter of non-oily POME particles (d_{eq}). The d_{eq} can be determined by the following equation [21]:

$$d_{eq} = d_{mean} * \varphi \qquad (16)$$

where φ, is a shape factor. In this study aspect ratio (AR) will be adopted as a shape factor. The mean particle diameter (d_{mean}) obtained from sieve process. The values of AR and d_{mean} where shown in Table V.

IV. CONCLUSION

This study proved that the relationship investigated by Richardson and Zaki (1954) between the free and hindered settling velocity for spherical particles can be adopted after modifying the index *n* to become suitable for non-spherical irregular non-oily POME particles. Furthermore, the study showed that the index *n* is a function of the particle Reynolds number (Re_p) and the ratio of POME particle diameter to the settling column diameter (λ). In this respect, this study found that the index *n* values were 4.65 for non-spherical POME particles that have sizes less than 63 µm. However, for other particles sizes that are greater than 63 µm, the index *n* values decreased with increasing their Re_p.

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