# Empirical Correlation of Pressure Drop of Porous Media in Darcy- Forchheimer Regime

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***Abstract*— The Fluid flow in porous media has been often modeled using Darcy's Law and Forchheimer's forms. The present paper studied the effects of low and high flow rate to determine if they may experience nonlinearity due to non-Darcy flow.The general experimental and mathematical models to evaluate the pressure drop for five types of porous media (sand, silt, porcilinaite, gravel and composed media) were achieved. The theoretical part of the present paper consists of an attempt to propose equations for estimating directly the pressure drop based on performing regression analysis on test results through the use of the trail version 9 of data Fit software of Oakdale Engineering computer programs.The proposed model for pressure drop of fluid flow in porous media is expressed as a function of tortuosity, porosity, pore particle and fluid properties was evaluated and proposed expressions showed good agreement with the experimental results.**

***Keywords- Darcy flow, Forchheimer's forms, regression analysis, propose equations.***

## I. SIGNIFICANT OF RESEARCH

The significant of this study is correlation empirical equation to evaluate the pressure drop and find the relationship between the friction factor and tortuosity

## II. INTRODUCTION

Flow studies through porous media are important in various applications in civil engineering, for example flows through rockfill dams or in aquifers [1]. Generally, flow data have been analyzed using Darcy's law which relates for sufficiently low flow the macroscopic velocity V and the pressure gradient as:

$$-\frac{dp}{dL} = \frac{\mu}{k} \cdot V \qquad (1)$$

where, dL a segment [m] along which a pressure drop dp [Pa] occurs, k is permeability or intrinsic permeability of the porous media, which depends solely on properties of the solid media. k has a dimension of area, or m2 in SI units. But the more convenient and traditional unit is the Darcy [26].

Many experimenters have attempted to use Reynolds concept to determine the upper limit of the validity of Darcy's law by Franzini [12],Hubbert [15], Scheidegger [20], Irmay[16], Ward [25], Sunada[22], Wright[27],Fair[11],Beavers[6], Ahmed[2], Bear[5] and MacDonald [18] had worked to show that Forchheimer's nonlinear equation could be derived from the first principles beginning with the Navier-Stokes equation. Recently, modeling progresses of flows within unconsolidated, granular media rely mostly on experimental works using homogeneous, spherical, artificial media (Comiti and Renaud[9]. In civil engineering, these studies are applied to the study of internal flows within earth and rock structures Martin[19]; Shih[21];Hansen [14]; Burchartet al.[8]; Hamilton[13];Wahyudi[24];Bingjun et al.[7]and to the problems of similarities of flow parameters in centrifuged geotechnical small-scale models, within which very large hydraulic gradients are often found (Babendreier[4]; Burchart[8]; Khalifa et al.[17].

The additional pressure drop due to inertial losses is primarily due to the acceleration and deceleration effects of the fluid as it travels through the tortuous flow path of the porous media. The total pressure drop is thus given by Forchheimer's empirical flow model stated traditionally as;

$$\frac{dH}{dx} = \frac{\mu}{k} v + \beta \rho v^2 \qquad (2)$$

The inertial resistance β(1/m) depends on the geometry of the solid medium, Tortuosity, permeability and porosity is frequently correlated to the flow parameters in porous media. To describe high-velocity flow in fracture sit is an important to understand the fracture geometry and mechanisms that generates high-velocity pressure loss[1].

Ergun [10] proposed empirical approximately equation to estimate the pressure drop and considered to be satisfactory over the range of flow rates encountered in packed bed as:

$$\frac{\Delta H}{\Delta L} = 150 \; \frac{\mu}{\rho g} \frac{(1-n)^2}{n^3} \frac{1}{d_s^2} V + 1.75 \; \frac{1}{g} \frac{(1-n)}{n^3} \frac{1}{d_s} V^2 \qquad (3)$$

Trussel and Chang [23] developed an empirical relationship for laminar flow regime in the cylindrical pores as:

$$\frac{\Delta H}{\Delta L} = \frac{32 \mu \tau}{\rho \, g} \frac{\tau^2}{d_p^2 n} V \qquad (4)$$

WU Jin-Sui et al. [28] proposed a model which was expressed as a function of tortuosity, porosity, resistance coefficient, and fluid properties. The total pressure drop is the sum of the viscous energy loss and kinetic energy losses along the flow paths, Where, $\zeta$ is the resistancecoefficient:

$$\frac{\Delta H}{\Delta L} = \frac{72 \mu \tau}{d_p^2} \frac{(1-n)^2}{n^3} V + \frac{3 \rho \zeta}{4 d_p} \frac{(1-n)}{n^3} V^2 \qquad (5)$$

$$\zeta = \left(1 - \frac{1}{d_{pore}^2}\right)^2 + 1.432$$

$$d_{pore} = \frac{1}{1-\sqrt{1-n}} \qquad (6)$$

From above we can confirmed the eq. 3 is expensively used

## III. Experimental Program

### *A. Materials*

Five types of porous media were test. The first four samples consisted of (sand, Gravel, Silt and Porcilinaite) porous media and one was composed by mixing four porous media with equal ratios (composed media), as shown in plate (1).

Porcilinaite Gravel Composed Silt Sand

Plate 1 Types of Porous Media

| Materials | Partical size distribution (ASTM) effevtive size d10 mm |
|---|---|
| Porcilinate | 8 |
| Gravel | 4.8 |
| Sand | 0.39 |
| Silt | 0.16 |
| Composed | 0.28 |

### *B. Sample Preparation.*

The sand Gravel and Porcilinaite samples were carefully washed with distilled water in order to purify them from any chemical impurities. Then samples were dried in the oven at 105 °C for 48 hours.

### *C. Sieve Analysis*

Grain-size Distribution Analysis: Figs.(1) and (2) below show the results of the particle size distribution analyses of the five media samples studied. To further analyze the distribution of the particles and to help classify the samples, the test results were then plotted on graphs to obtain the grain-size distribution curves for each sample. From the grain-size distribution curves, samples were classified according to ASTM (D422).

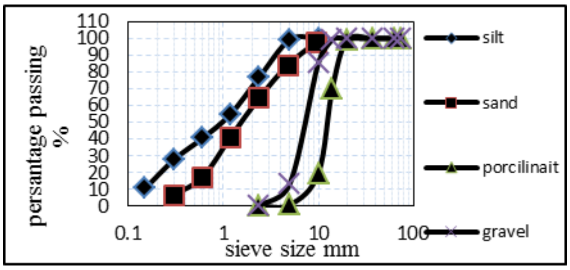


Fig. 1 Sieve Size Distribution Curve of the Samples Tested.

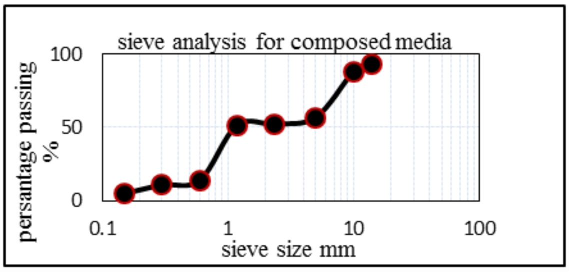


Fig. 2 Sieve Size Distribution Curve of Composite Porous Media.

### *D. Experimental set- up*

The experimental set-up, the schematic diagram and the photograph of which were presented in Fig.(3) and plate (2), respectively which consist of both a hydraulic device and a measurement device.

The hydraulic device consists of a five horizontal PVC pipe (6 in diameter and 6 m length) filled with five types of porous media. The system was supplied water from a 2000 liter tank. Intensity of inflow was controlled by valve, and measured by a flowmeter. In order to determine the flow rate, we calibrated the flowmeter using a stop watch to measure the time it took to fill the 3.5 inch-diameter column to (1liter) height. Pressures were measured by pressure gauges in five points equally distributed along the pipe with a distance 1.5 m. Water temperature is measured by a thermometer. Basic experimental parameters are summarized in Table (1).

Table (1) Basic Parameters of the Flow Set

| No | Parameter | Symbol | Value |
|---|---|---|---|
| 1 | average water temperature | T | 14 CO |
| 2 | density of water | ρ | 999.22 ( kg/m3) |
| 3 | Dynamic viscosity coefficient | μ | 1.173e-3 (kg/(m· s)) |
| 4 | gravity acceleration | g | 9.81 m/S2 |

During the experiment, were done for different values of volumetric inflow Q, Based on these values and the pipe cross-section area, mean velocity were calculated as divided discharge over a cross sectional area of pipe. Pressure differences Δp between two measurements points as well as Reynolds numbers Re were computed from equation

$$R_e = \frac{d_p \, \rho V}{\mu \, (1-n)} \qquad (7)$$

where, $dp = d_{10}$

Different values of velocity (v), during the experimental are summarized in Table (2).

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## IV. ANALYTICAL MODELS

Numerical simulations reproducing the experiment were performed with use of regression analysis on test results through the use of the trail version 9 of data Fit software of Oakdale Engineering computer programs

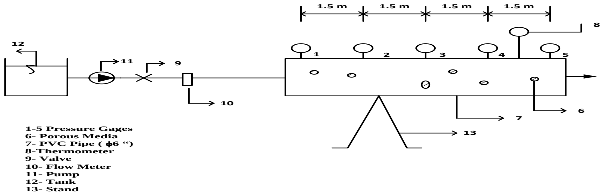


Fig. 3 Pressure Mode

Plate (2) Pressure Model

### *A. Darcy Model*

The pressure drop is a function of five variables as shown in the following expression:

$$\frac{\Delta p}{L} = f\ (n, S, \tau, v, d_{10})$$

Where, n, s, τ,v, $d_{10}$ are porosity, specific surface area, tortuosity, velocity and effective diameter.

The constants were calculated by regression analysis using data points obtained from the experiment work. Based on experimental results obtained from this paper, the empirical equation of pressure drop can be correlated as:

$$\frac{\Delta p}{L} = 17.98\ \frac{\mu}{g\rho}\frac{(1-n)^2}{n^3}\frac{1}{d_{10}^2}S^2\tau^{9.94} \quad \text{Sand} \quad (8)$$

$$\frac{\Delta p}{L} = 7.17\ \frac{\mu}{g\rho}\frac{(1-n)^2}{n^3}\frac{1}{d_{10}^2}S^2\tau^{8.71}V \quad \text{Silt} \quad (9)$$

$$\frac{\Delta p}{L} = 17.634\ \frac{\mu}{g\rho}\frac{(1-n)^2}{n^3}\frac{1}{d_{10}^2}S^2\tau^{8.38}V \quad \text{Composed} \quad (10)$$

Figs.(4) to (6) shows the experimental values versus proposed values of pressure drop using equation (8) to (10).

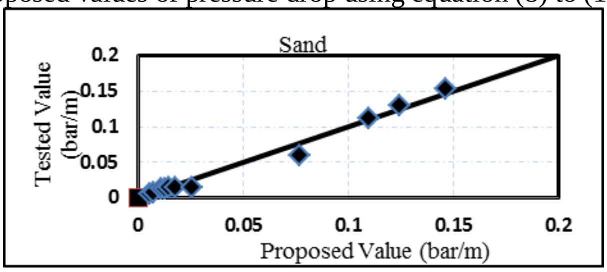


Fig. 4 Experimental Values Versus Proposed Values of Pressure Drop

Table 2 Set of Velocity.

| Media | Range of velocity (m/s) | Re | Applied model |
|---|---|---|---|
| **Sand** | **0.000235-0.0094** | **0.354- 9.448** | **Darcy's regime applicability** |
| **Silt** | **0.000786-0.00613** | **0.471- 3.673** | **Darcy's regime applicability** |
| **Composite medias** | **0.00252-0.00786** | **2.452- 7.649** | **Darcy's regime applicability** |
| **Gravel** | **0.00282-0.0674** | **26.85-100.14** | **Forchheimer's regime applicability** |
| **Porcilinaite** | **0.00282-0.0674** | **77.097-600.76** | **Turbulent regime applicability** |

Compares of experimental results with the proposed Equation (8 to 10) show that the proposed Equation seems too reasonable to estimate the pressure drop of porous media for linear model as shown in Figs.(4 to 6).

Figs.(7 to 9) compare the proposed model prediction by Equation (8)-(10) with these by Ergun Equation (3), WU Jin Sui and YIN Shang (2008) Equation(5), Trussel and Chang Equation (1999) Equation(4) and experiment data in the present study.

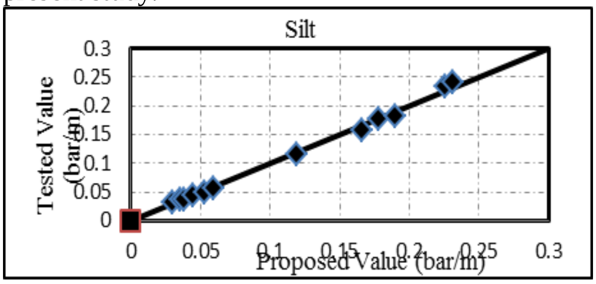


Fig. 5 Experimental Values versus Proposed Values of Pressure Drop

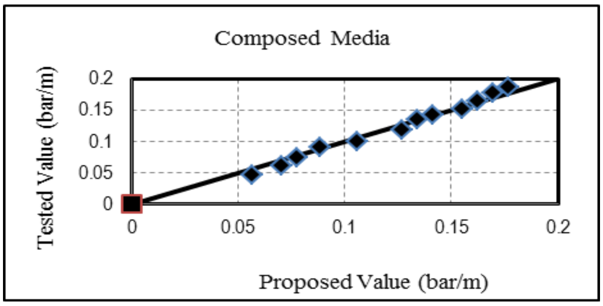


Fig. 6 Experimental Values versus Proposed Values of Pressure Drop.

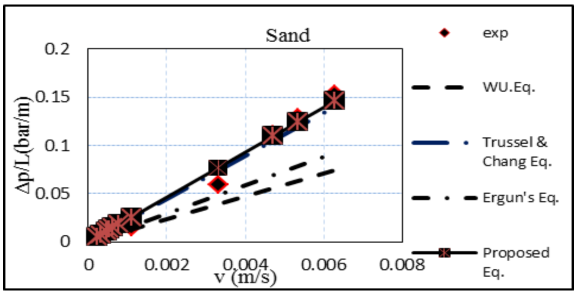


Fig. 7 Comparison of Pressure Drop for the Present Model, Experiment Data, Ergun Model, WU Jin and YIN Shang Model and Trussel and Chang Model.

Fig.(7) shows for sand media that good agreement is obtained among the proposed model predictions by Equation. (8), experiment data and Trussel and Chang model. While for silt media good agreement was obtained among the proposed model with the experiment data and WU Jin and YIN Shang model as shown in Fig.(8).

From Fig. (9) it can be seen that the proposed models predictions by Equation. (9) and Equation. (10) for non-uniform media (composed media) was concurrent with the experiment data and Ergun model.

It was observed that all proposed expressions properly to estimate the pressure drop for linear models for all media study in the present paper.

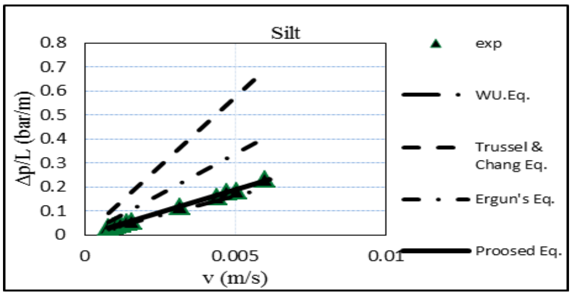


Fig. 8 Comparison of Pressure Drop for the Present Model, Experiment Data, Ergun Model, WU Jin and YIN Shang Model and Trussel and Chang Model.

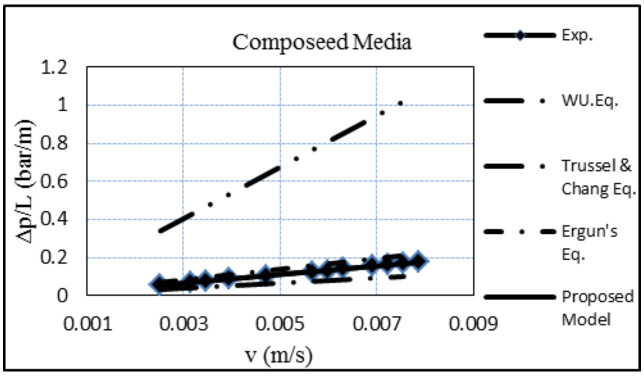


Fig. 9 Comparison of Pressure Drop for the Present Model, Experiment Data, Ergun Model, WU Jin and YIN Shang Model and Trussel and Chang Model.

### B. *Forchheimer and Turbulent Models*

The pressure drop is a function of four variables as shown in the following expression:

$$\frac{\Delta p}{L} = f\ (n\ ,, \tau, v, d_{10})$$

The constants were calculated by regression analysis using data points obtained from the experiment work.

Based on experimental results obtained from this paper, the empirical equation of pressure drop can be correlated as:

$$\frac{\Delta p}{L} = 46.2\frac{\mu}{g\rho}\frac{(1-n)^2}{n^3}\frac{1}{d_{10}^2}S^2\tau\ V + 1.615\ \frac{1}{g}\frac{(1-n)}{n^3}\frac{1}{d_{10}}\tau^{2.495}v^2 \quad \text{gravel} \quad (11)$$

for turbulent model, the empirical equation of pressure drop can be corrected as:

$$\frac{\Delta p}{L} = 2.454\ \frac{1}{g}\frac{(1-n)}{n^3}\frac{1}{d_{10}}\tau^{2.315}v^2 \quad \text{porcilinaite(12)}$$

Comparisons with experimental result indicate that the proposed Equation (11 and 12) properly estimate the pressure drop for non- linear model of porous media as shown in Figs.(10 and 11).

Figs.(12) and (13) compares the proposed model prediction by Equation (11) and (12) with these by Ergun model Equation. (4), WU Jin Sui and YIN Shang (2008)

Figs.(12) and (13) show for gravel and porcilinaite media that good agreement are obtained among the proposed models predictions by Equation. (11) and Equation. (12), Ergun Equation., Jin-Sui Equation and experiment data.

Equation (5) and experiment data, which were measured in the present paper.

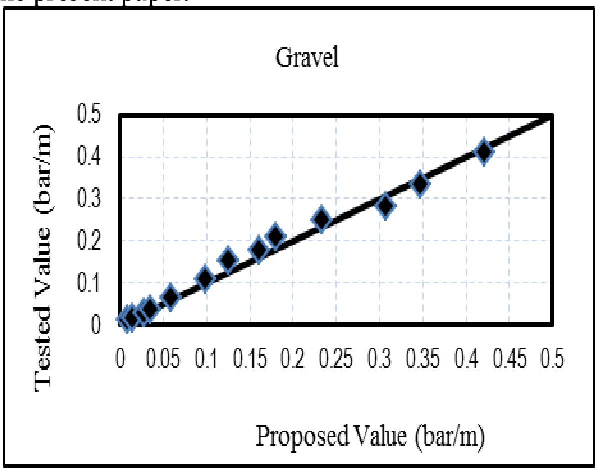


Fig. 10 Experimental Values versus Proposed Values of Pressure Drop

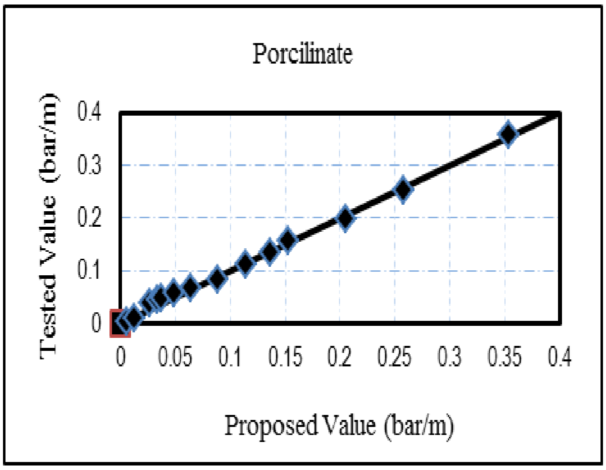


Fig. 11 Experimental Values versus Proposed Values of Pressure Drop

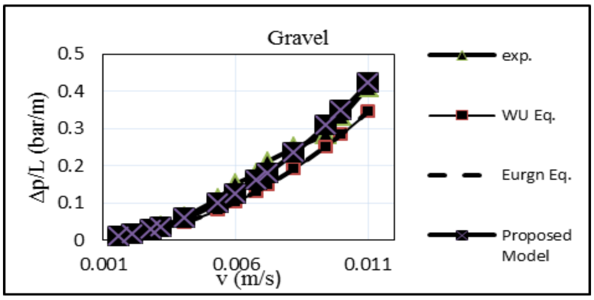


Fig. 12 A Comparison Among the Present Model, Experiment Data, Ergun Model andWU Jin Sui and YIN Shang Model.

## V. Result and Discussions

As shown in Figs.(7) to (9) a linear relationship between pressure drop and the flow velocity in Darcy regime become polynomial relationship in Forchheimer regime. For sand, silt, and composed media. The Reynolds number value for this measurement is closed to a limit value of Re=10, which is compatible to the range of applicability of Darcy's law.

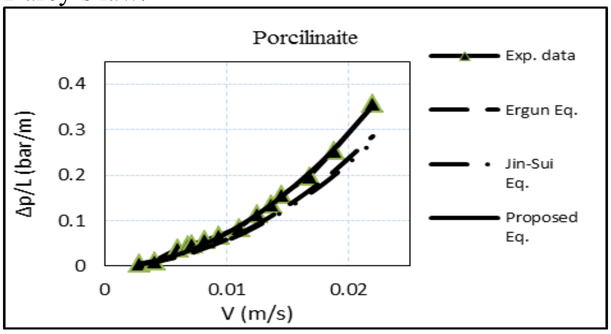


Fig. 13 A Comparison Among the Present Model, Experiment Data, Ergun Model andWU Jin Sui and YIN Shang Model

The structure of porous media such as particle size, porosity, grain distribution and surface area of porous media plays a major role in pressure drop. It was found that an increase in particle diameter causes a decrease in pressure drop. This is due to the fact that when the pore diameter increases the void between particle increases and this leads to a decrease in the resistance to fluid flow.

At high rates the pressure drop exceeds and it is found to be proportional to the square of the velocity as shown in Figs.(12) to (13). It was found that an increase in particle diameter causes a decrease in pressure drop. The pressure drop values in the gravel are twice than that in the porcilinaite rock because of diameter particle of porcilinaite is greater than that of the gravel.

## VI.Conclusions

1- The linear models of Darcy's law are widely applicable than Forchheimer model because of their computational simplicity in comparison with their nonlinear model. There exists a range of filtration velocities where linear models provide a satisfactory approximation and may be successfully applied without accuracy scarified.

2- The proposed model for pressure drop of fluid flow in porous media is expressed as a function of tortuosity, porosity, pore particle and fluid properties properly estimate the pressure drop for linear and nonlinear model

3- An increase in pore diameter causes a decrease in pressure drop, this is due to the fact when the pore diameter increases the void between practical increases, and this leads to a decrease in the resistance to fluid flow.

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