

# Estimates for Linear hardening and Effective Yield Stress of Nano-Porous Media with Non-Uniform Distribution of Voids

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**Abstract.** This study aims at studying first; the relation between the effective yield stress of perfectly plastic matrix and spherical and elliptical voids, 33 numerical tests carried out with different effective yield stress of the matrix (different materials), so as to estimate the relationship between the effective yield stress of the porous media and perfect plastic matrix. The relationship between the effective yield stress of the matrix and porous media is estimated by a mathematical equation with less than 5% error. Using finite element method, and within the representative volume element, the porous materials are modeled according to its microstructure.

And second; the proposed model tested numerically and in the case of linear hardening and another relation is estimated between the linear hardening coefficient of the matrix and the porous media. 66 numerical tests carried out, based on the finite elements numerical tests with three dimensional representatives volume elements; these mathematical equations are proposed so as to predict the macroscopic behavior of the porous media, depending on the solid matrix properties. These numerical simulations show that the porosity volume fraction can has a strong effect on the overall effective yield stress and linear hardening coefficient.

**Keywords-component; heterogeneous materials, nonlinear homogenization techniques, finite element modelling, porous materials.**

## I. INTRODUCTION

During the last 30 years, several numerical models proposed and extended, attempting to explain the relationship between the porous media and the matrix phase properties. Perhaps the popular model for porous plastic solids is proposed by [1]. More recently, porous media models are extended to large strains, first in the framework of hypoelastoplasticity after the work of [2] for single-phase materials, and using hyperelastoplasticity. To the author knowledge, a partially saturated model has been put forward only in the framework of hypoelastoplasticity, based on an updated Lagrangian approach, Eulerian strain rate tensor and Jaumann stress rate. In the present paper, a partially saturated porous media model is developed in the framework of hyperelastoplasticity, extending the previous work of [3]. Conditions of partial saturation are of

importance in engineering practice because many porous materials are in this natural state or can reach this state during deformations. Some simple examples can be found in soils or in concrete and in biological tissues, which can contain air or other gases in the pores together with liquids. For instance, this is the case of the soil zones above the free surface, or the case of deep reservoirs of hydrocarbon gas. The partially saturated state can also be reached during the deformation due for instance, to earthquake in an earth dam or during the particular case of strain localization of dense sands under globally undrained conditions, where negative water pressures are measured and cavitation of the pore water was observed see [3] and [4].

A computational homogenization method was used by [5] to estimate the effective elastic-plastic response of particulate composites. The method is based on the computations of small limited volumes of fixed size extracted from a larger one and containing different realizations of the random microstructure. They showed that the elementary volume element containing one centered inclusion, even widely used in the literature, represents a minor bound of the real mechanical response.

About porous materials, there are a lot of studies. Some of them deal about the plasticity models. The classical plasticity model of Gurson [1] contains the porosity as internal variable and it was considered perfect plastic in the matrix of porous materials. [6] and [7] proposed the extended yield of Gurson type model, so called GTN model, with the new parameters  $q_i$ , with  $i=1,2,3$ , to adjust their experiments. It most frequently used. In the GTN model, isotropic hardening is accounted for the flow stress of the matrix material  $\sigma_Y$ . While, in [7], they proposed the modification of  $f^*$  as a piecewise linear function of the initial porosity  $f$ . The value of  $f^*$  is considered an increase of porosity due to the coalescence of the voids and then they extended the critical void volume fraction  $f_c$ .

Studying hardening properties for porous materials also is one of the interested subjects by some other authors. [8] and [9] proposed the linear hardening into Gurson model. [10] presented the comparison between Gurson model and unit cell model, indicating the void growth during isotropic and

kinematic hardening for porous materials using the same principal of [9]. Another analytical function, for spherical void in a spherical volume element, proposed by [11], considering incompressible isotropic and kinematic hardening matrix material. They proposed an extension of GTN model. Later, the same principle is used by [12]; [13] and [14]. They studied the evaluation of the yield surface of porous materials, using elementary volume element containing one void, using finite element method. A three - dimensional unit cell, with one void, is simulated by [15] considering void growth in ductile materials. [16] compared their calculation of a unit cell with Gurson and the porous metal plasticity model (LPD) with considering hardening, using continuum damage mechanics. [17] discussed the general effects of porosity on the mechanical behavior (tensile, fatigue) of a specific composite. Focused on the discussed studies the calculation of a unit cell and some evaluation of damage under cyclic loading with hardening, and they discussed the loading stories.

The interested point in this study is to propose analytical equation which is able to describe macroscopic hardening properties for porous materials under simple tension. A plasticity formulation can have a variety of yield criteria, time independent or time dependant flow, isotropic hardening. The used isotropic hardening model for matrix phase here is linear isotropic hardening as in equation 1. And for non linear hardening, the most commonly equations used for isotropic hardening are power law (Ludwik law), equation 2, and the exponential law (Voce law), equation 3; which were equations used recently by several authors, see e.g. [18], [2], [19], [16] [20], [12], [13], [21], [22], [23], [24] and [25]. Linear and non linear evolutions of isotropic hardening, for the matrix phase only, can be calculated according to the equations below, power law or exponential law, respectively:

$$\sigma_Y = \sigma_o + H\varepsilon^p \quad (1)$$

$$\sigma_Y = \sigma_o + H(\varepsilon^p)^b \quad (2)$$

$$\sigma_Y = \sigma_o + Q(1 - e^{-b\varepsilon^p}) \quad (3)$$

For porous materials, we considered that the matrix phase is obey Voce law of isotropic hardening, where  $\sigma_Y$  is the macroscopic uni-axial stress,  $Q$  corresponds to the amplitude of the exponential function (saturation hardening, positive or negative).  $b$  denoted the coefficient of decay (rate of saturation hardening).  $H$  is material parameter (plastic hardening modulus),  $\varepsilon^p$  represents the microscopic equivalent plastic strain and  $\sigma_o$  the initial yield stress of the pure matrix.  $Q$  and  $b$  can be adjusted to a given material. The sum of  $Q$  and  $\sigma_o$  is denoted as the maximal yield surface radius in deviatory stress space.  $b$  governs the shape of the hardening curve between the lower  $\sigma_o$  and the upper ( $\sigma_o + Q$ ) bounds. Increasing the value of  $b$  leads to reach the upper bound faster, for  $Q = 0$  or  $b = 0$ , isotropic hardening does not occur. The variable  $p$  is the cumulated plastic strain. Isotropic hardening is applied to the yield as follows :

$$\sigma_{iso} = \sigma_{eq} - \sigma_Y \quad (4)$$

$\sigma_{eq}$  has the same scalar measure of the von Mises stress equivalent, equation 6.

We considered here that the material used is a plastic porous with porosity  $f$ . The deviatory von Mises equivalent stress  $\Sigma_{eq}$  and the hydrostatic part of the macroscopic stress tensor  $\Sigma_m$  are :

$$\Sigma_m = \frac{1}{3}tr(\bar{\sigma}) = \frac{\sigma_{11} + \sigma_{22} + \sigma_{33}}{3} \quad (5)$$

$$\Sigma_{eq} = \sqrt{\frac{(\sigma_{11} - \sigma_{22})^2 + (\sigma_{11} - \sigma_{33})^2 + (\sigma_{22} - \sigma_{33})^2}{2}} \quad (6)$$

The equivalent plastic strain  $\varepsilon^p$  can be determined according to the equation below, see e.g. [26], [27], [28], [16], [29], [14] and [12], [13]:

$$\varepsilon^p = \int_{t=0}^t \dot{\varepsilon}^p dt \quad (7)$$

$$\dot{\varepsilon}^p = \sqrt{\frac{2}{3} \dot{\varepsilon}_{ij}^p \dot{\varepsilon}_{ij}^p} \quad (8)$$

$t$  represents time and  $\dot{\varepsilon}^p$  is plastic strain rate.

Several authors used the same principal of Voce law for hardening and they proposed the material parameters ( $Q$ ,  $b$ ) for different materials and composites, see e.g. [30], [20], [21], [22], [23], [31], [24], [25] and [32]. From equation 3, the relation between the equivalent stress and the equivalent strain or multi component strain hardening model can be expressed as:

$$\Sigma_{eq} = \sigma_o + \sum_i Q_i (1 - e^{-b_i \varepsilon^p}) \quad (9)$$

see e.g. [33] and [30].

In this study, we propose a new general analytical model which it can be used for different porous materials and different porosities, using numerical calculations of finite element methods for a three dimensional representative volume elements. The motivation of proposing this analytical model is, as we mentioned, there are a lot of studies about isotropic hardening for materials and composites as well as the mentioned papers, see e.g. [34], [35], [36], [37], [38] and [39], but there is few studies about isotropic of porous materials which also an important subject. We studied this subject and we proposed an analytical model for isotropic porous hardening.

The purpose of this paper is to introduce an analytical model to predict macroscopic behavior for porous materials by considering isotropic hardening in the matrix phase. Through performing numerical finite element results of a sufficiently large three dimensional representative volume element of porous material. The considered volume contains 200

randomly distributed identical spherical voids. An elementary volume element, with one centered spherical void, is also used in the finite element simulation. The proposed analytical model is an extension of the exponential law for isotropic hardening of pure matrix. We proposed this macroscopic model based on the numerical calculations of 3D RVE finite element method. The numerical results are obtained in case of simple tension tests for different porosities considering isotropic hardening in the matrix phase. In the future work we consider cyclic loading (tension-compression) and we test the model under triaxial loading.

The present paper is organized as follows, in section 2, we present the investigated microstructure and the computational method and the proposed analytical model. The results are presented and discussed in section 3. Some concluding remarks are given in section 4.

## II. THE REPRESENTATIVE VOLUME ELEMENTS

First, Representative volume elements (Generate of RVE). The representative volume element (RVE) with 22%, 12% of spherical voids and 15% of elliptical voids have been used and total of 3 groups of numerical tests (33 tests for each group) were carried out, the RVE model illustrated in Fig 1. three dimensional RVE was used with a fine meshing.

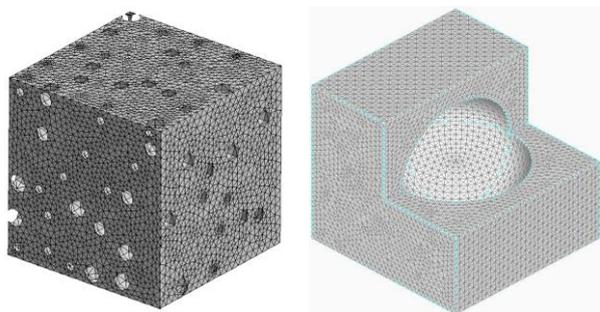


Figure 1. Example RVE of porous material with 22% porous volume fractions

## III. NUMERICAL EXPERIMENTS

### A. Finite Element Modeling

We presented the procedure of creating 3D RVE with FE mesh for porous materials. Indicating materials properties to the components, which are the essential steps toward presenting the modeling and the numerical analysis. The procedure for defining boundary conditions is explained. The mesh size and the steps of the calculation for different microstructures and different volume fractions explained in details in this section.

### B. Microstructure and mechanical behavior

Materials considered in this study are made of plastic matrix containing identical spherical voids randomly

distributed. The microstructure is plastic matrix with random or periodically distribution of voids, through studying RVE, containing 200 randomly distributed voids or elementary volume element contains one centered void respectively, see Fig 1. The Young's modulus of the matrix  $E = 1550MPa$  and the Poisson ratio  $\nu = 0.4$  are assumed. The saturation hardening  $Q = 200$  and the rate of saturation hardening  $b = 10$ , which supposed to obey the von Mises plasticity model with initial yield stress  $\sigma_o = 245MPa$  is performed, considering that there is isotropic hardening, and there is no overlapping between the voids, while the random distribution of 200 voids in an RVE represents random microstructure and elementary volume element which contains only one void represents a periodic microstructure. Three groups of tests are performed. The values of matrix and voids parameters of the RVE that used in all numerical experiments well explained in table 1, 2 and 3.

Test no.	Parameter				
	Perfect plasticity phase (matrix) No hardening			void phase, inclusions spherical and elliptical	
	Elastic Modulus $E_{em}$ (MPa)	Poisson's ratio $\nu_{em}$	Effective Yield Stress (MPa)	Elastic Modulus $E_{ev}$ (MPa)	Poisson's ratio $\nu_A$
0	1550	0.4	245	0.001	0.3
1	1550	0.4	250	0.001	0.3
2	1550	0.4	255	0.001	0.3
3	1550	0.4	260	0.001	0.3
4	1550	0.4	265	0.001	0.3
5	1550	0.4	270	0.001	0.3
6	1550	0.4	275	0.001	0.3
7	1550	0.4	280	0.001	0.3
8	1550	0.4	285	0.001	0.3
9	1550	0.4	290	0.001	0.3
10	1550	0.4	290	1550	0.4

TABLE I. VALUE OF MATRIX AND VOIDS PARAMETERS FOR POROUS MEDIA GROUP NO. 1, (33) TESTS, 12%, 22% SPHERICAL, AND 15% ELLIPTICAL VOIDS. EFFECTIVE YIELD STRESS IS VARIABLE.

Test no.	Parameter					
	Plasticity phase (matrix) Linear hardening phase				void phase, inclusions spherical and elliptical	
	Elastic Modulus $E_{em}$ (MPa)	Poisson's ratio $\nu_{em}$	Effective Yield Stress (MPa)	linear hardening coefficient	Elastic Modulus $E_{ev}$ (MPa)	Poisson's ratio $\nu_A$
0	1550	0.4	245	2000	0.001	0.3
1	1550	0.4	245	2100	0.001	0.3
2	1550	0.4	245	2200	0.001	0.3
3	1550	0.4	245	2300	0.001	0.3
4	1550	0.4	245	2400	0.001	0.3
5	1550	0.4	245	2500	0.001	0.3
6	1550	0.4	245	2600	0.001	0.3
7	1550	0.4	245	2700	0.001	0.3
8	1550	0.4	245	2800	0.001	0.3
9	1550	0.4	245	2900	0.001	0.3
10	1550	0.4	245	2900	1550	0.4

TABLE II. VALUE OF MATRIX AND VOIDS PARAMETERS FOR POROUS MEDIA GROUP NO. 2, (33) TESTS, 12%, 22% SPHERICAL, AND 15% ELLIPTICAL VOIDS. LINEAR HARDENING COEFFICIENT IS VARIABLE.

Test no.	Parameter					
	Plasticity phase (matrix) linear hardening phase				void phase, inclusions spherical and elliptical	
	Elastic Modulus $E_{em}$ (MPa)	Poisson's ratio $\nu_{em}$	Effective Yield Stress (MPa)	linear hardening coefficient	Elastic Modulus $E_{ev}$ (MPa)	Poisson's ratio $\nu_{ev}$
0	1550	0.4	245	2500	0.001	0.3
1	1550	0.4	250	2500	0.001	0.3
2	1550	0.4	255	2500	0.001	0.3
3	1550	0.4	260	2500	0.001	0.3
4	1550	0.4	265	2500	0.001	0.3
5	1550	0.4	270	2500	0.001	0.3
6	1550	0.4	275	2500	0.001	0.3
7	1550	0.4	280	2500	0.001	0.3
8	1550	0.4	285	2500	0.001	0.3
9	1550	0.4	290	2500	0.001	0.3
10	1550	0.4	290	2500	1550	0.4

TABLE III. VALUE OF MATRIX AND VOIDS PARAMETERS FOR POROUS MEDIA GROUP NO. 1, (33) TESTS, 12%, 22% SPHERICAL, AND 15% ELLIPTICAL VOIDS. EFFECTIVE YIELD STRESS IS VARIABLE.

C. Boundary conditions

Another issue for numerical tests after generating the RVE is the boundary conditions (BC); choosing the correct BC leads to the correct results. The BC for the total numerical tests in this work were taken as simple as possible; by fixing a plane in the loading direction (example plane 1), and fixing one point in all directions (example point *o*) to prevent the displacement in any direction and the other plane remain as free, With fixing the second point in one perpendicular directions to loading direction, (example point *b* in *Z* direction) so as to prevent rotation [5]. This procedure well explained in Fig. 2.

For a uni-axial tensile loading in the *x* direction, prescribed these conditions as follows:

$$\begin{aligned}
 u\{\text{plane } (x=0, y, z)\} &= 0 & v\{\text{point } O(0,0,0)\} &= 0 \\
 u\{\text{plane } (x=l, y, z)\} &= \delta & w\{\text{point } O(0,0,0)\} &= 0 \\
 v\{\text{point } A(0,0,l)\} &= 0 & w\{\text{point } B(0,l,0)\} &= 0
 \end{aligned}
 \tag{10}$$

in which, *u*, *v* and *w* are the applied displacements in the *x*, *y* and *z* directions, *l* is the RVE length and *d* is the prescribed displacement.

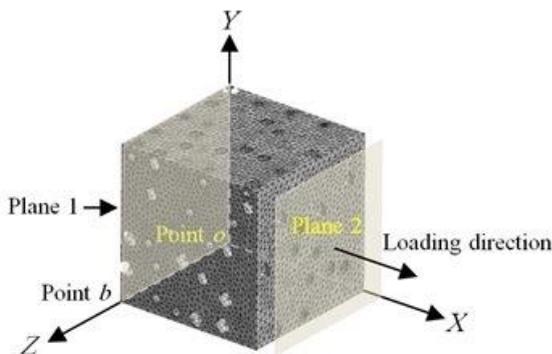


Figure 2. Boundary conditions description of the RVE that used in all the experiment

D. Materials and approach

Before you begin In the numerical experiments for porous media with randomly distributed spherical voids, two phases (spherical voids and perfect plastic matrix) and the properties of the materials are indicated in table 1, 2 and 3. So as to examine the efficiency of the approach 10 numerical tests have been carried out for each volume fraction 12%, and 22% spherical voids and 15% elliptical voids. Very good results and the same relationships were obtained from all volume fractions.

IV. RESULTS AND DISCUSSION

A. Relationship between the effective yield stress of the perfect plastic matrix and the porous media

Estimating the relationship between the effective yield stress of the perfect plastic matrix and porous media is very important. For 10 different effective yield stress matrix with same volume fractions of voids 12%, and 22%, we obtained the following mathematical linear relationship: if  $\sigma_{ep}$  = porous media effective yield stress,  $\sigma_{em}$  = matrix effective yield stress, and  $\nu_{fm}$  = matrix volume fraction. All the tests were done with fixing all parameters for the voids and matrix while only  $\sigma_{em}$  effective yield stress of the matrix has been changed in each test and  $\sigma_{ep}$ , porous media effective yield stress has been found in each time. Based on the above numerical tests the following mathematical relationship has been found:

$$\sigma_{ep} = \sigma_{em} \times \nu_{fm}
 \tag{11}$$

Noted that about less than 5% error, as explained in Fig 3, 4 and 5.

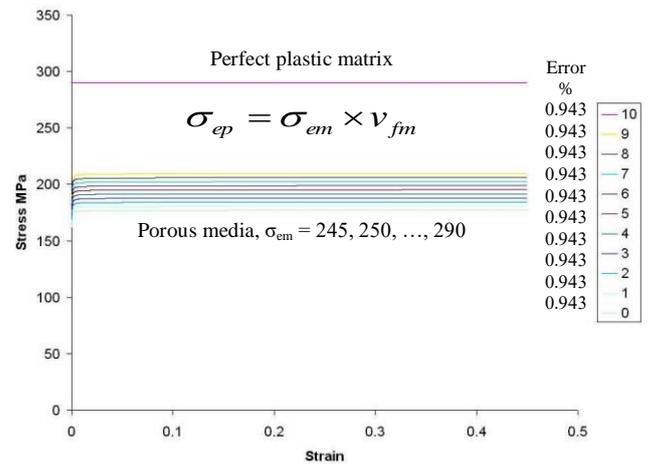


Figure 3. Relation between effective yield stress of the perfect plastic matrix and porous media for 22% spherical voids.

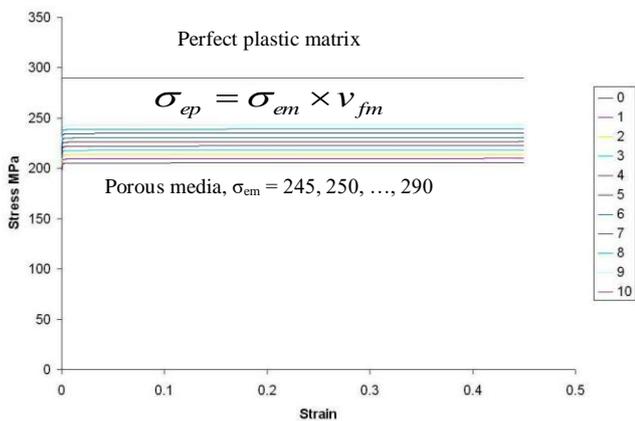


Figure 4. Relation between effective yield stress of the perfect plastic matrix and porous media for 12% spherical voids

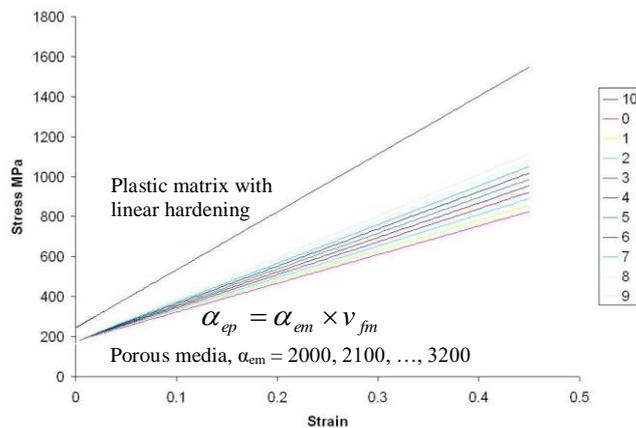


Figure 6. Relation between linear hardening coefficient of the matrix and porous media for 22% spherical voids..

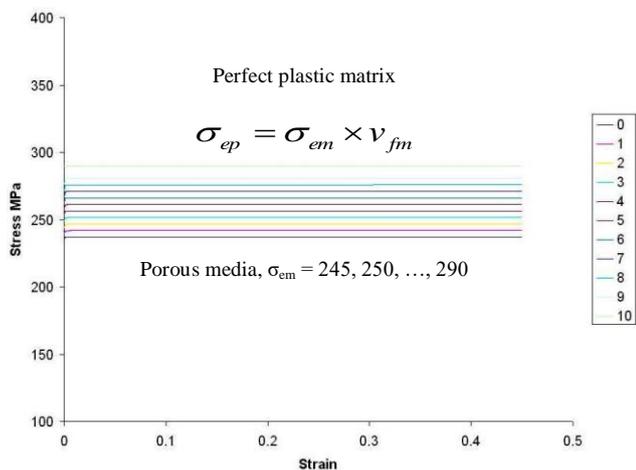


Figure 5. Relation between effective yield stress of the perfect plastic matrix and porous media for 15% elliptical voids.

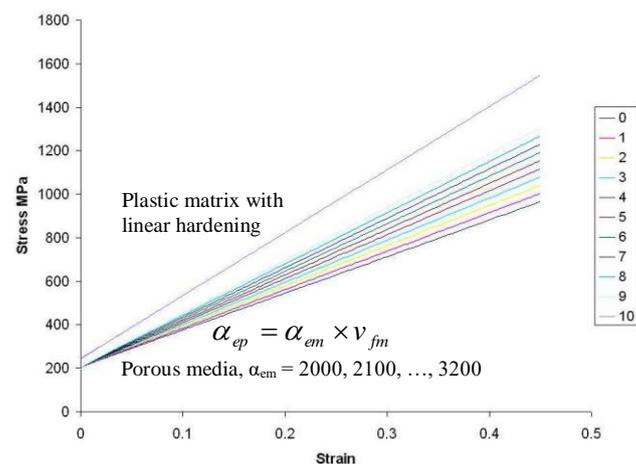


Figure 7. Relation between linear hardening coefficient of the matrix and porous media for 12% spherical voids.

**B. Relationship between the linear hardening coefficient of the perfect plastic matrix and the porous media**

Estimating the relationship between the linear hardening coefficient of the perfect plastic matrix and porous media is very important. For 10 different linear hardening coefficient for the matrix with same volume fractions of voids 12%, and 22%, we obtained the following mathematical linear relationship: if  $\alpha_{ep}$  = porous media linear hardening coefficient,  $\alpha_{em}$  = matrix linear hardening coefficient, and  $v_{fm}$  = matrix volume fraction. All the tests were done with fixing all parameters for the voids and matrix while only  $\alpha_{em}$  linear hardening coefficient of the matrix has been changed in each test and  $\alpha_{ep}$ , porous media linear hardening coefficient has been found in each time. Based on the above numerical tests the following mathematical relationship has been found see Fig 6, 7 and 8:

$$\alpha_{ep} = \alpha_{em} \times v_{fm} \tag{12}$$

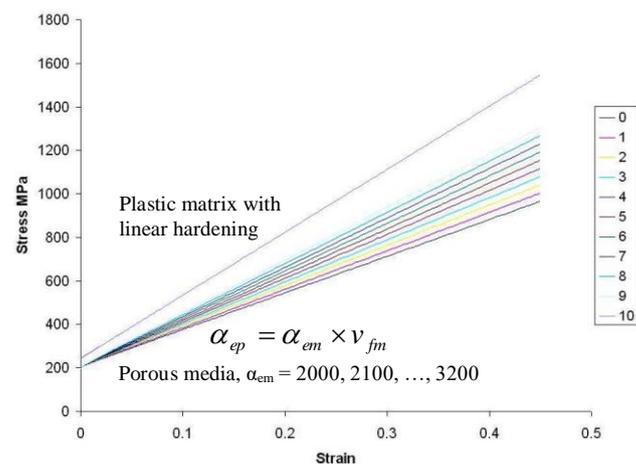


Figure 8. Relation between linear hardening coefficient of the matrix and porous media for 15% elliptical voids.

C. Relationship between the effective yield stress of the plastic matrix and the porous media during linear hardening.

Estimating the relationship between the effective yield stress of the plastic matrix and porous media is very important during the linear hardening. For 10 different effective yield stress matrix with same volume fractions of voids 12%, and 22%, and the same linear hardening coefficient we obtained the following mathematical linear relationship: if  $\sigma_{ep}$  = porous media effective yield stress,  $\sigma_{em}$  = matrix effective yield stress, and  $v_{fm}$  = matrix volume fraction. All the tests were done with fixing all parameters for the voids and matrix while only  $\sigma_{em}$  effective yield stress of the matrix has been changed in each test and  $\sigma_{ep}$ , porous media effective yield stress has been found in each time. Based on the above numerical tests the following mathematical relationship has been found:

$$\sigma_{ep} = \sigma_{em} \times v_{fm} \tag{13}$$

During the linear hardening operations.

We noted that there is a significant relations between the effective yield stress of the perfect plastic matrix and porous media with linear hardening. Noted that about less than 5% error, as explained in Fig 9, 10 and 11.

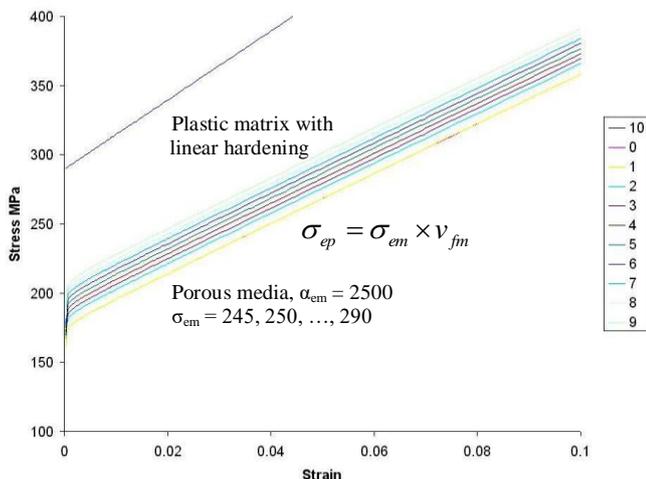


Figure 9. Relation between effective yield stress of the plastic matrix and porous media for 22% spherical voids, with linear hardening operation.

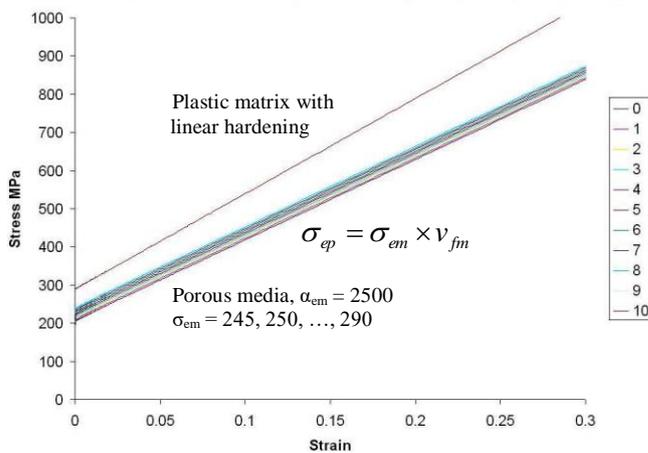


Figure 10. Relation between effective yield stress of the plastic matrix and porous media for 12% spherical voids, with linear hardening operation.

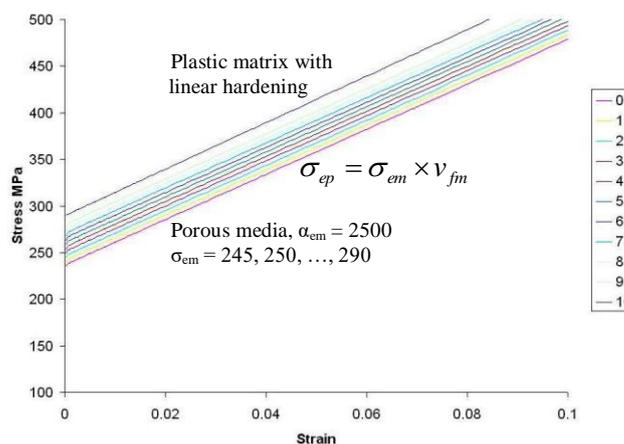


Figure 11. Fig 5c Relation between effective yield stress of the plastic matrix and porous media for 15% elliptical voids, with linear hardening operation.

V. CONCLUSIONS

A three dimensional micromechanical finite element formulation was implemented for the efficient computation of effective yield stress of a volume fraction 12% and 22% volume fractions of spherical voids and volume fraction of 15% for elliptical voids for porous media. In order to model and analyze the relationship between the linear hardening coefficients and effective yield stress for each matrix and porous media, a cubic three dimensional representative volume was used, the nano-spherical voids and nano-elliptical voids were simulated as a second phase while the matrix was regarded as perfect plastic phase and with linear hardening in the second group of tests.

Thee mathematical models are proposed here to predict the properties of porous media based on the void content and the properties of the matrix in the case of perfect plastic matrix and with linear hardening.

The main advantages of the present models are; the relation between the linear hardening coefficient and effective yield stress of matrix and porous media can be determined, which were well explained in this paper.

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