Comparison between Euler Bernoulli Beam Theorem and ANSYS 14 for Undamped Aluminum Cantilever Beam

Mr. IYD EQQAB MAREE
Erbil Polytechnic Universality
Erbil Technical Engineering College
Erbil, Iraq
aiyed67@yahoo.com

***Abstract*—The beam is one of the fundamental elements of an engineering structure. It finds use in varied structural applications. Moreover, structures like airplane wings and stabilizer, helicopter rotor blades, spacecraft antennae, flexible satellites, gun barrels, robot arms, high rise buildings, long span bridges, and subsystems of more complex structures can be modeled as a beam like slender member. These types of structure goes to crack, fatigue and damages because of high vibration frequency and amplitude, also some time in case of structures have a thickness more than 6mm and during high vibration frequencies goes to damage because the structure move horizontally to a side.**

Keywords-component; formatting; style; styling; insert (key words)

1. Introduction

THERE ARE TWO METHODS TO PREDICT THE BEAM VIBRATORY RESPONSE FOR UNDAMPED CANTILEVER BEAM, FIRST METHOD IS EULER-BERNOULLI BEAM THEORY AND SECOND METHOD IS FINITE ELEMENT MODELS ANSYS14.

2.1 First Method: Euler Bernoulli beam theory

The Euler-Bernoulli beam theory is commonly used for predicting the vibratory response of beams. Within its limitations, the Euler-Bernoulli theory predicts beam response very well. Since an understanding of beam theory and boundary conditions are important for this research, a review of Euler-Bernoulli beam theory is provided. The detail of assumptions behind the Euler-Bernoulli beam as stated by Inman [2] are listed below, in which the beam is considered to be:

1. Uniform along its span, or length, and slender
2. Composed of a linear, homogeneous, isotropic elastic material without axial loads
3. Such that plane sections remain plane
4. Such that, the plane of symmetry of the beam is also the plane of vibration so that rotation and translation are decoupled
5. Such that, rotary inertia and shear deformation can be neglected.

The Euler-Bernoulli beam equation of motion can be written as:

$$f(x,t) = \rho A(x)\frac{\partial^2 \omega(x,t)}{\partial t^2} + \frac{\partial^2}{\partial x^2}\left[EI(x)\frac{\partial^2 \omega(x,t)}{\partial x^2}\right] \quad \text{..........(1.1)}$$

The boundary conditions require that each end of the beam have specified moment ($\frac{\partial^2 \omega(x,t)}{\partial x^2}$), or slope ($\frac{\partial \omega(x,t)}{\partial t}$) and shear force ($\frac{\partial^3 \omega(x,t)}{\partial t^3}$) or displacement $\omega(x,t)$. The boundary conditions that are evaluated in this research are the clamped-free (cantilever).

The cantilever boundary condition requires that $\omega(x,t) = \frac{\partial \omega}{\partial x} = 0$ at $x = 0$ location and ($\frac{\partial^2 \omega(x,t)}{\partial x^2} = \frac{\partial^3 \omega(x,t)}{\partial t^3} = 0$) at $x = L$.

The frequencies for the undamped Euler-Bernoulli beam can be identified through an equation 1.2 [3].

$$\omega_n = \beta_n{}^2 \sqrt{\frac{EI}{\rho A}} \quad \text{.........(1.2)}$$

Where

$\rho =$ mass density

$E =$ Modulus of elasticity

β_n can be found from the Eigen values for each of the boundary conditions as shown in table 1.1) [82].

$$I = \frac{bd^3}{12} \quad \text{......(1.3)}$$

Where b and d are the breadth and width of the beam cross-section as shown in the figure 1.1.

Table 1.1 is a list of the corresponding Eigen values for Euler-Bernoulli for different types of beam clamped-free (cantilever beam), Free - Free, Pinned - Pinned and Clamped - Clamped.

2.2 Second Method: Finite element analysis (ANSYS 14)

For design purposes, engineers are more concerned with the ability to model in finite element software packages than to understand sixth-order differential equations. For this reason, finite element software packages such as ANSYS 14 specifically, eight nodded (SOLID183) will be used to model damped systems, providing feedback both to the designer and the researcher on the effectiveness of each model and its corresponding parameters.The output results from ANSYS 14 (SOLID183), modal frequency values (natural frequency) and mode vibration shape for the undamped aluminum cantilever beams with constant length 20cm and constant width 2.5cm .

3. Predicting the vibratory response of Undamped Aluminum Cantilever Beam

3.1 Euler Bernoulli Beam Theory

Euler-Bernoulli beam theory is commonly used for predicting the vibratory response of beams. Within its limitations, the Euler Bernoulli theory predicts beam response very well. Since an understanding of beam theory and boundary conditions is important to this research, a review of Euler Bernoulli beam theory is provided.

From Table 1.1 and equation 1.2, we can calculate the undamped natural frequency for aluminum specimens with constant length of 20cm and width of 2.5cm, with different thicknesses (4mm, 6mm and 8mm) for the first five modes of vibration.

The calculated undamped natural frequency by using Euler theory for aluminum beam can be shown in Table 1.2 and Figure 1.2. From the Figure 1.2 showed that the undamped natural frequency of the undamped aluminum cantilever beam increased with increasing the thickness of the beam for constant length and width, this may be due to increasing the thickness of the beam the moment of inertia increased and this was leaded to increase the undamped natural frequency.

3.2 Finite Element Models

Finite element software packages such as ANSYS 14 was used to model damped systems. It was provided feedback both to the designer and the researcher on the effectiveness of each model and its corresponding parameters. The results of finite element software packages provided in this research were obtained by using two-dimensional solid structural elements in ANSYS14 specifically eight nodded (SOLID183), and each element having two translational degrees of freedom per node. The output results from ANSYS14 (SOLID183) for undamped aluminum cantilever beam specimen (4mm thickness), modal frequency values and mode shape of vibration with constant length 20cm and width 2.5cm for the first five modes are shown in Figure 1.3. Figure 1.3 showed that the movement of undamped aluminum cantilever beam 4mm thickness for first five modes of vibration is vertical, while Figure 1.4 show that the fifth mode for undamped aluminum cantilever beam specimens thickness (6mm and 8mm) moved horizontal to the sides and this type of movement lead to crack, fatigue and damage the undamped cantilever beam. This is due to a rotary inertia and shear deflection effects are often significant in the lateral deflection of short beams. The significance decreases as the ratio of the radius of gyration of the beam cross section to the beam length becomes small compared to unity [4]. Shear deflection effects and rotary inertia are activated in the stiffness matrices of ANSYS14 beam elements by including a nonzero shear deflection constant in the real constant (SOLID183) list for that element type.

3.3. Comparison between Euler Bernoulli Beam Theorem and ANSYS 14 for Undamped Cantilever Beam

To find the degree of coincidence between Euler beam theorem and ANSYS14 for undamped cantilever beam, a comparison was made between the values of modal frequency and vibration modes shape obtained from the two methods as shown in Figures 1.5. Figures 1.5 showed that the values of modal frequency for three undamped aluminum cantilever beam specimens obtained from Euler Bernoulli beam theory closely matched the modal frequency obtained from ANSYS14 (SOLID183) for specimens 4mm thickness and for first five modes of vibration and the percentage error between the two methods was very low. Figure 1.5 showed that the values of modal frequency obtained from Euler Bernoulli beam theory were in agreement with the modal frequency obtained from ANSYS14 (SOLID183) for two undamped aluminum cantilever beam specimens (6mm and 8mm thickness). The percentage error between the two methods for the first four modes of vibration varied from 0.03% to 1.8 %, while in fifth vibration mode for undamped aluminum cantilever beam specimen (8mm thickness), the percentage of error between them was 31% as shown in Table 1.3. This is because in the fifth vibration mode, as shown in ANSYS14 (Figure 1.4 b and c), the movement of specimen was horizontally to the sides for the same reason as mentioned before. While, in the Euler Bernoulli beam theory, rotary inertia and shear deflection are neglected and this lead to the specimen moved vertically up and down.

4. Conclusion

1.The modal frequency values for undamped aluminum cantilever beam specimens with thickness of 4mm, 6mm and 8mm obtained from Euler Bernoulli beam theory were in good agreement with the modal frequency obtained from ANSYS14 for the first four vibration mode shape. The percentage of error between the two methods was very low for the first four vibration modes, while the percentage error in the fifth vibration mode for undamped cantilever beam specimens for thickness 6mm and 8mm was high. This increment in percent error was because in the fifth mode of vibration the movement of specimen was horizontally to side and this is due to a rotary inertia and shear deflection effects. While,in the Euler Bernoulli beam theory, rotary inertia and shear deflection are neglected and this lead to the specimen moved vertically up and down.

2.The Euler Bernoulli beam theory was very accurate method to calculate the modal frequency (natural frequency) for undamped cantilever beam for thickness 4mm, while it can be used for thickness 6mm and 8mm just for first four vibration mode with coincidence.

3. The undamped natural frequency of the undamped aluminum cantilever beam increased with increasing the thickness of the beam for constant length and width and this was due to effect of the moment of inertia.
4. The vibration movement of undamped cantilever beam for 4mm thickness was vertically up and down for first five vibration modes, while the fifth mode for undamped cantilever beam specimens (6mm and 8mm thickness) moved horizontally to a side. This horizontal movement to a side for undamped cantilever beam would lead to crack, fatigue and fracture of the beam and it must be prevented.

5. Reference

[1] BRIAN DRUNYON, The Influence of boundary condition and aspect ratio on approximate solutions for constrained layer damping treatment on beams and plates, Wright State University, 2001.
[2] D. J. Inman, Engineering vibration. New Jersey: Prentice-Hall, Inc., second ed., 2000.
[3] Dr. S.P. NIGAM," Study of vibration characteristics of cantilever beams of different materials ", THAPAR UNIVERSITY PATIALA-147004, INDIA, July 2012.
[4]http:/www.researchgate.net/publictopics.PolicPostFileLoader.html?id=52f4f746d3df3e1f498b45ee&key=5046352f4f746b790e.

Figures and Tables:

Table 1.1: Euler-Bernoulli Eigen values [1].

Bending Mode	Clamped-Free	Free-Free	Pinned - Pinned	Clamped-Clamped
	$\beta_n L$	$\beta_n L$	$\beta_n L$	$\beta_n L$
1	1.87510407	4.730040740	π	4.73004074
2	4.69409113	7.85320462	2π	7.85320462
3	7.85475744	10.9956078	3π	10.9956079
4	10.99554073	14.1371655	4π	14.1371655
5	14.137101	17.2787597	5π	17.2787597

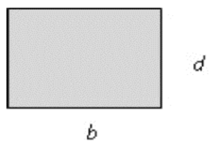


Figure 1.1: Cross-section of the cantilever beam

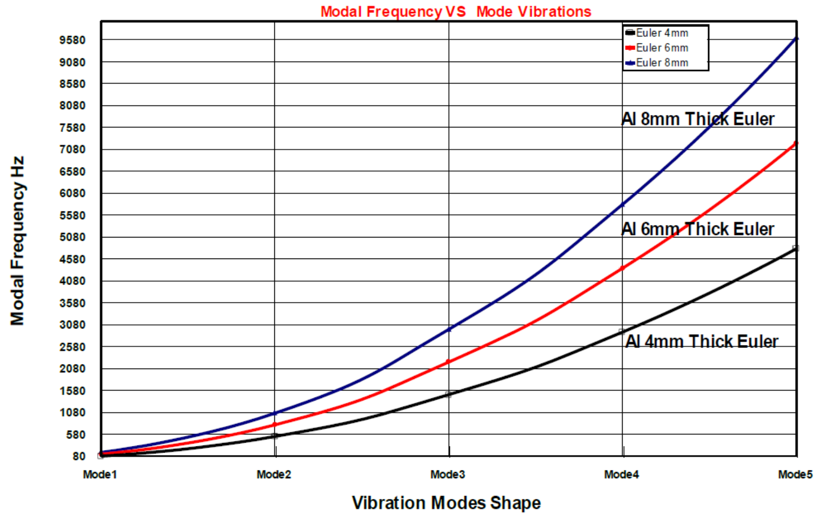


Figure 1.2: Comparison between modal frequencies for three undamped aluminum cantilever beam thickess by using Euler theorm method

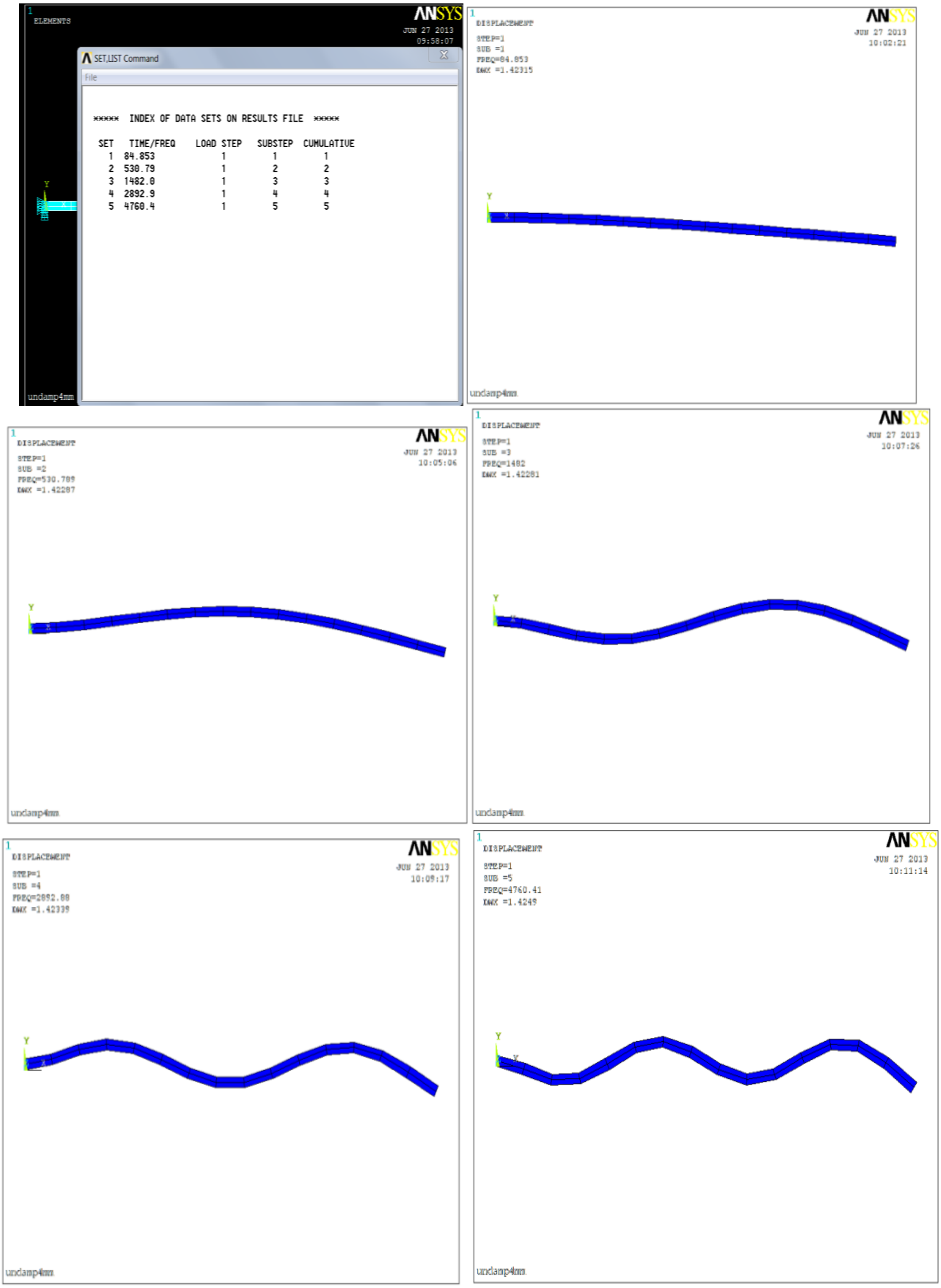
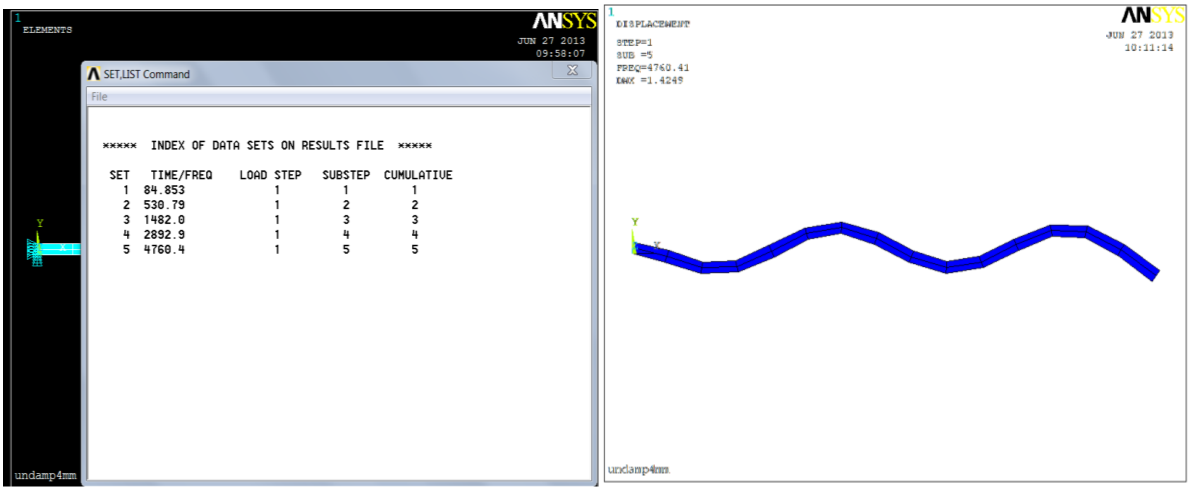
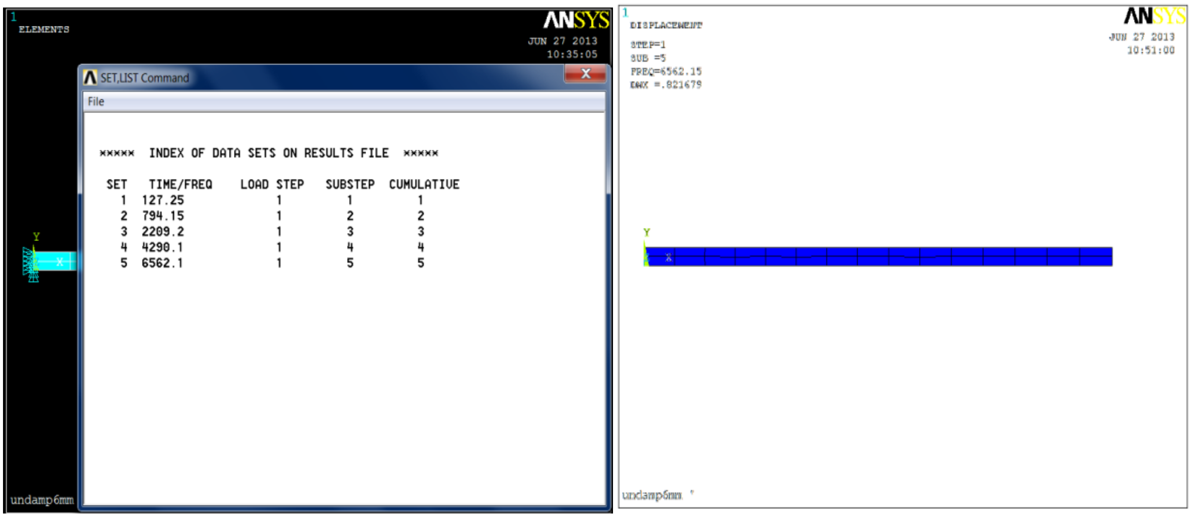


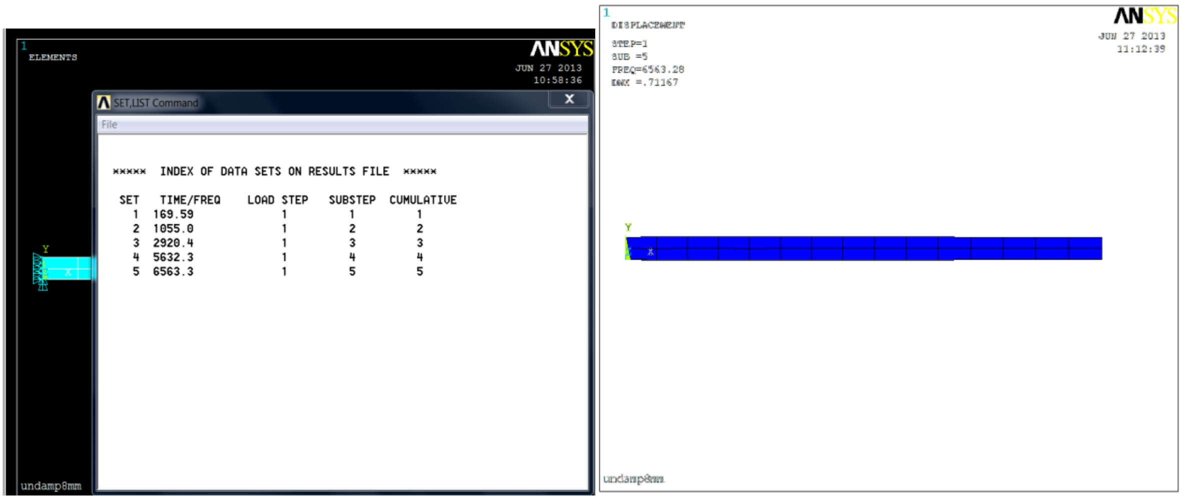
Figure 1.3: Modal frequency values and vibration modes shape for the first five modes for specimen undamped aluminum cantilever 4mm thickness



(a) 4mm thickness fifth mode shape of vibration moved vertically up and down



(b) 6mm thickness fifth mode shape of vibration moved horizontally to the sides



(c) 8mm thickness fifth mode shape of vibration moved horizontally to the sides

Figure 1.4: Modal frequency values and vibration mode shape for the fifth mode for specimen aluminum undamped cantilever with different thickness

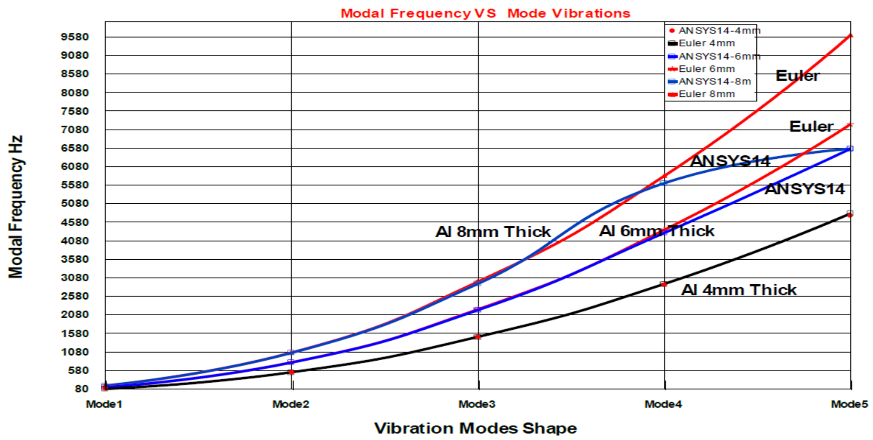


Figure 1.5: Comparison between the Euler theorem and ANSYS14 for undamped aluminum cantilever beam with different thicknesses

Table 1.2: Description of specimens and calculation of modal frequency for three undamped aluminum cantilever beams by using Euler beam theorem for constants, width b = 0.025m , length=0.2m , $\rho = 2470\ \mathrm{kg/m^3}$ and $E = 68\mathrm{GPa}$

For first mode $\beta_n L = 1.87510407$

Undamped specimens	Thickness(a) m	I $\times 10^{-10}$ m^4	A m^2	β_n	$\left[\omega_n = \beta_n{}^2\sqrt{\frac{EI}{\rho A}}\right]$ rad/s	f Hz
4mm	0.004	1.333	0.0001	9.3755	532.5	84.74
6mm	0.006	4.5	0.00015	9.3755	798.8	127.1
8mm	0.008	10.66	0.0002	9.3755	1064	169

For second mode $\beta_n L = 4.69409113$

Undamped specimens	Thickness(a) m	I$\times 10^{-10}$ m^4	A m^2	β_n	ω_n rad/s	f Hz
4mm	0.004	1.333	0.0001	23.47	3336	531
6mm	0.006	4.5	0.00015	23.47	5006	796
8mm	0.008	10.66	0.0002	23.47	6672	1062

For third mode $\beta_n L = 7.85475744$

Undamped specimens	Thickness(a) m	I$\times 0^{-10}$ m^4	A m^2	β_n	ω_n rad/s	f Hz
4mm	0.004	1.333	0.0001	39.273	9342	1486
6mm	0.006	4.5	0.00015	39.273	14015	2230
8mm	0.008	10.66	0.0002	39.273	18687	2975

For fourth mode $\beta_n L = 10.99554073$

Undamped specimens	Thickness(a)m	I× 0^{-10} m^4	A m^2	β_n	ω_n rad/s	f Hz
4mm	0.004	1.333	0.0001	54.977	18307	2914
6mm	0.006	4.5	0.00015	54.977	27465	4371
8mm	0.008	10.66	0.0002	54.977	36612	5827

For fifth mode $\beta_n L = 14.1370$

Undamped specimens	Thickness(a) m	I× 10^{-10} m^4	A m^2	β_n	ω_n rad/s	f Hz
4mm	0.004	1.333	0.0001	70.685	30263	4816
6mm	0.006	4.5	0.00015	70.685	45402	7225
8mm	0.008	10.66	0.0002	70.685	60523	9632

Table 1.3: Degree of coinciding between Euler theorem and ANSYS 14 for undamped aluminum cantilever beam with different thicknesses (4mm,6mm and 8mm)

	Modes	**ANSYS 14 Hz**	**Euler theory Hz**	**Errors %**
Undamped aluminum specimen 4mm	First	84.853	84.74	0.1
	Second	530.79	531	0.03
	Third	1482	1486	0.2
	Fourth	2892.9	2914	0.7
	Fifth	4760.6	4816	1.1
Undamped aluminum specimen 6mm	**Modes**	**ANSYS 14 Hz**	**Euler Theory Hz**	**Errors %**
	First	127.25	127.1	0.1
	Second	794.15	796	0.2
	Third	2209.2	2230	0.9
	Fourth	4290.1	4371	1.8
	Fifth	6562.1	7225	9.1
Undamped aluminum specimen 8mm	**Modes**	**ANSYS 14 Hz**	**Euler Theory Hz**	**Errors %**
	First	169.59	169	0.3
	Second	1055	1062	0.6
	Third	2920.4	2975	1.8
	Fourth	5632.3	5827	3.3
	Fifth	6563.3	9632	31