The Effect of ADC Quantization Error by Rounding to Nearest Integer Bit

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Abstract — **This paper considers Analog to Digital Conversion ADC by Rounding to Nearest Integer Bit method to eliminate the error produced in digitization processing. Also, a statistical method has been applied to predict the required effect of the quantization error in ADC processing. The adopted technique is a statistical approach to find out the effects of the quantization error on the performance of an ADC converter. It is assumed that the quantization error of the sine signal is random, sequential uniformly distributed and additive to the original signal. The excess digits are reduced through the use of Rounding method. Presenting a statistical method is to predict the required effect of the quantization error using rounding to the nearest integer bit, to meet the prescribed specifications. The methods are demonstrated and calculated by MATLAB and by STATGRAPHIC program. Finally, the results show that as the number of bits increased in digital words, the experimental and theoretical numerical value of Signal to Quantization Noise Ratio (SQNR) is very similar and the value of SQNR is 6dB for each bit increased.**

Keywords- Analog Digital Conversion, SQNR, Quantization error.

I. INTRODUCTION

Throughout the analog signal to digital signal converter (ADC), the amplitude of the analog signal to be converted has infinite precision. The continuous amplitude must be converted into digital data with the finite precision, which called is the quantization [1]. The quantization is an approximate process, the input values to the quantizer are the real value and the output values represent approximated values [2], [8].

Therefore, the error introduced in representing the continuous value signal by a finite set of the discrete value levels is called quantization error or noise. The quantization noise or error is a sequence (xe), defined as the difference between the quantized value (xq) and the actual sample value of the sine signal (x), { $xe = xq - x$ }. The treatment of this quantization error (xe) is carried out using rounding method which assigns each sample signal to the nearest quantization level [2],[3].

In this work, the effect of the quantization error on the sine signal using rounding method is reviewed in section (II). The adopted statistical program is discussed in section (III). Finally a comparison between the averages of the two set values of the Error-mean quantization using MATLAB and Stat-graphics programs are given in section (IV).

II. LITERATURE REVIEW

The subject of quantization error reduction has been studied by many researchers in terms of some parameters. The quantization noise induced by $\Delta\sum$ modulation has been suppressed more using a technique to reduce the frequency step [10]. In an echo-cancelling full-duplex transceiver, the effect of quantization noise cancelling removes the DAC quantization noise [11]. In here, the point is focusing on the performance of an ADC converter according to quantization error. In addition, using MATLAB and STATGRAPHIC is to demonstrate clear results in the method of Rounding to nearest integer bit.

III. EFFECT OF THE QUANTIZATION ERROR ON THE SINE SIGNAL USING ROUNDING METHOD

First using the MATLAB program below to find out the Signal-to-quantization-Noise –Ratio (SQNR) in dB, and error-mean quantization using (Rounding) [7],[8], then finding and comparing between Mean Deviation (MD) and Standard Deviation (SD) of the quantized error in case of accuracy.

By using above MATLAB program for different number of bits (n=2, 4, 6, 8, 10, 12, 14, 16), resulted of getting table (1)

Table 1. The results of variable n.

No. of Bits(n)	Experimental SQNR Rounding	Theoretical SQNR Rounding	(Error-Mean) Quantization Using (Rounding) method
2	14.5930	13.8000	0.1071
4	26.2010	25.8400	0.0289
6	37.8781	37.8800	0.0077
8	49.7767	49.9200	0.0020
10	61.7435	61.9600	0.00049985
12	74.8614	74.0000	0.00010662
14	85.7604	86.0400	0.000032085
16	98.0608	98.0800	0.0000075907

Calculation of quantization steps (Δ) when number of bits n=2

$$\text{The quantization steps } (\Delta) = \frac{\text{Full Range (R)}}{\text{No.of levels (L)}} \qquad (1)$$

R (Full range) = 2* Amplitude = 2* 1 = 2

L (No. of levels) = $2^n = 2^2 = 4$ where (n=2)

Δ (quantization steps) = $\frac{2}{4} = 0.5$ for (n=2)

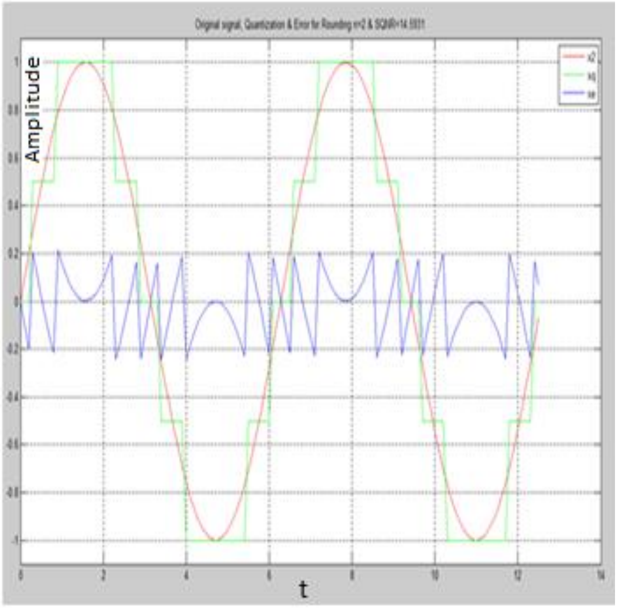

Figure 1. Original analog sine signal, Quantization and error using Rounding method for number of bits (n=2)

From the Fig. (1), the rang of quantization error using rounding method of the sine analog signal is symmetric about zero and not resulting from a reduction of number of significant bits as well as falls in the rang of all positive and negative numbers [3], [4], [5].

$$[-\mathbf{0.25} = -\frac{\mathbf{0.5}}{\mathbf{2}} = -\frac{\Delta}{2} \leq \boldsymbol{Quntization\ Error}(\boldsymbol{Rounding}) \leq \frac{\Delta}{2} = \frac{\mathbf{0.5}}{\mathbf{2}} = \mathbf{0.25}]$$

Similarly from the fig (2) the graphs of the original analog sine signal, Quantization and error using Rounding method for number of bits (n=2, 4, 6, 8, 10, 12, 14, 16), it shows that as the number of bits in the word increased resulted in reducing the quantization error. Figure (2) represents an original analog sine signal, Quantization and error using Rounding method for number of bits (n=2, 4, 6, 8, 10, 12, 14, 16). Comparing between Standard Deviation (SD) and mean deviations (MD) of the set values of the quantization error using Rounding method resulted to the conclusion. The standard Deviation of the quantization error will be always greater to Mean Deviation of the quantization error for the same signal, Standard deviation ≥ mean deviation [6].

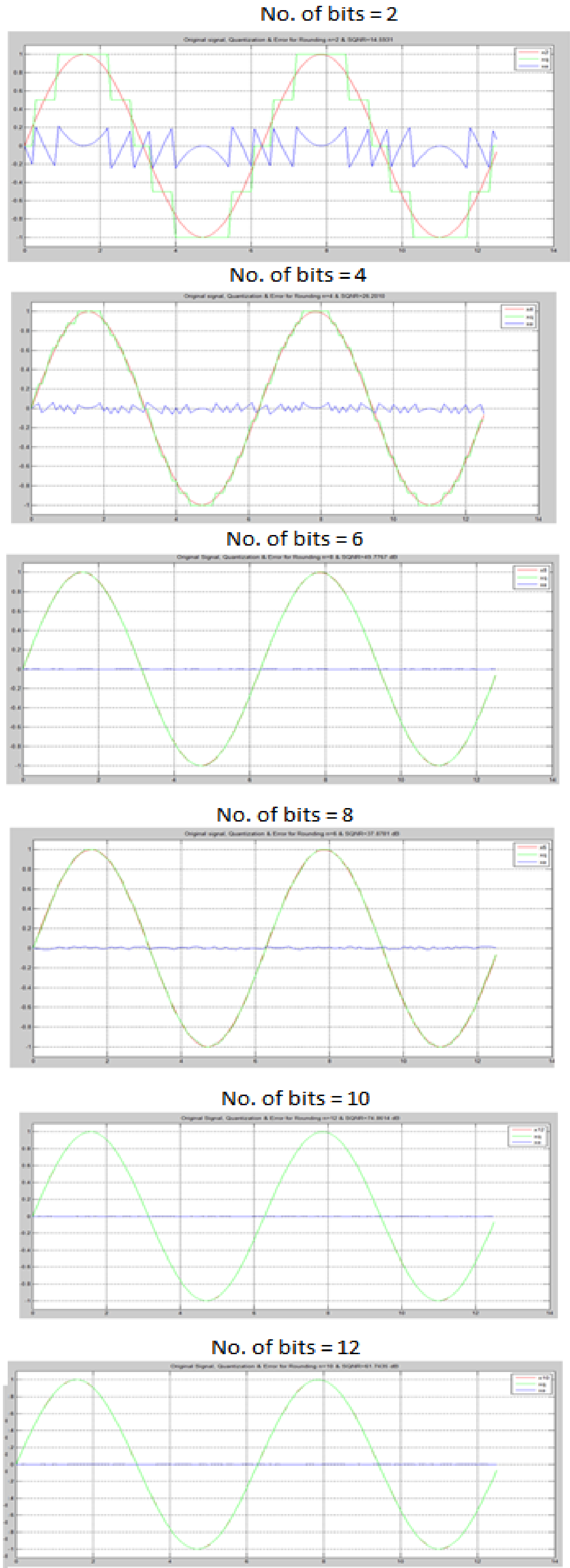

Figure 2. Original analog sine signal, Quantization and error using Rounding method for number of bits (n=2, 4, 6, 8, 10, 12, 14, 16)

Table 2. SD & MD of the set of values

n(Bits)	EMQ	Y'	(y-y')^2	|y-y'|
2	0.1071	0.018293268	0.007886636	0.088806732
4	0.0289	0.018293268	0.000112503	0.010606732
6	0.0077	0.018293268	0.000112217	0.010593268
8	0.002	0.018293268	0.000265471	0.016293268
10	0.00049985	0.018293268	0.000316606	0.017793418
12	0.00010662	0.018293268	0.000330754	0.018186648
14	0.000032085	0.018293268	0.000333471	0.018261183
16	7.5907E-06	0.018293268	0.000334366	0.018285677
Summation			0.009692023	0.198826926
SD = $\sqrt{\frac{(y-y')^2}{n-1}}$ $\sqrt{\frac{0.009692023}{n-1}}$=0.03720387388			MD=∑|y-y'|/n = 0.198826926/8 =0.02485336575	

Table (2) shows the numerical values of SQNR, for both theoretical and experimental, and Error_mean Quantization (EMQ) of the sine signal by using the Rounding method.

IV. Adopting a statistical program

(STATGRAPH) is used to compare and perform analysis between simple regressions. The output shows the results of fitting a linear model to describe the relationship between Errors mean Rounding and [1/ (No. of bits)^2].

The equation of the fitted model is Error means rounding = $-0.00282247 + 0.442382 * 1/(NO. of\ bits)^2$ (2)

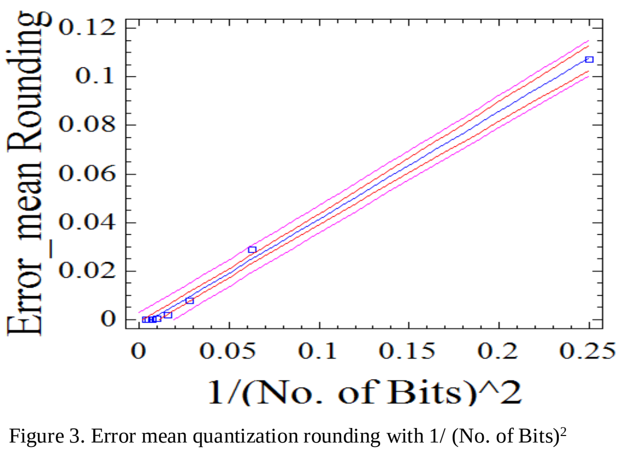

Figure 3. Error mean quantization rounding with 1/ (No. of Bits)^2

Dependent variable: Error means Rounding & Independent variable: 1/No. of Bits

Intercept = - 0.00282247 Slope = 0.442382

V. Comparison between the averages of the Error-mean quantization using MATLAB and Stat-graphic programs

Table 3. Compare between the averages of the Error-mean quantization using MATLAB and Stat-Graphics programs

NB	EMQMAT	EMQSGP
2	0.1071	0.10777303
4	0.0289	0.024826405
6	0.0077	0.009465575
8	0.0020	0.004089749
10	0.00049985	0.00160135
12	0.00010662	0.000249608
14	0.000032085	-0.000565437
16	0.0000075907	-0.001094415
Average	**0.018293268**	**0.018293233**

Where NB represents number of bits, EMQMAT represents (Error-mean)quantization using rounding method in MATLAB, and EMQSGP refers to (Error-mean) quantization (Stat-graphics program) that

EMQSGP=-0.00282247+0.442382*1/(No. of bits)^2

Also, from table (4) the values of error-mean quantization are determined for each number of bits. The experimental and theoretical Signal to quantization Noise Ratio (SQNR) of the sine signal using Rounding method are very similar, while the number of bits are inversely proportional to the quantization error and the rang of quantization error is symmetric about zero which not resulting from a reduction of number of significant bits as well as falls in the rang of all positive and negative numbers.

We observe that the quantization step after Rounding has a value ($\Delta = 2^{-n}$) and the maximum rounding error, therefore has a magnitude of ($\frac{2^{-n}}{2} = \frac{\Delta}{2}$) , as result the range of quantization error Rounding is given by

$$[-\frac{\Delta}{2} \leq Quantization\ Error(rounding) \leq \frac{\Delta}{2}]$$

Calculating the Standard deviation of the quantization error (SD) = 0.037215588 is always greater than mean absolute deviation (MD) = 0.02485336575, Standard deviation ≥ mean deviation. It means the measures of the deviations of the original observation (values) from their mean are greater than the mean deviation which measure of how much the values in the data set are likely to differ from their mean.

VI. Conclusion

By using statistical program (STATGRAPHICS PROGRAM), the R-Squared statistic indicates that the fitted model explains 99.71801544% of the total variation of dependent variable using Rounding, which means less than 1% or exactly (0.28198456%) of variations depends on the other factors which is not included in this study and The standard error of the estimate using Rounding method is 0.002134549133. In other words the precision of model increases as the standard Error of estimate decreases.

From the table 3 the average of Error-mean using stat graphics program is less than the average of Error-mean using MATLAB program by ($3.5x10^{-8}$) this means the differences are not significant between these two programs. And hence could be ignored.

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